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**APPLICATION OF A MULTI-OBJECTIVE EVOLUTIONARY ALGORITHM TO THE
SPACECRAFT STATIONKEEPING PROBLEM**

A Thesis in
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by
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ABSTRACT

Satellite operations are becoming an increasingly private industry, requiring increased profitability. Efficient and safe operation of satellites in orbit will ensure longer lasting and more profitable satellite services. This thesis focuses on the use of a multi-objective evolutionary algorithm to schedule the maneuvers of a hypothetical satellite operating at geosynchronous altitude, by seeking to minimize the propellant consumed through the execution of stationkeeping maneuvers and the time the satellite is displaced from its desired orbital plane. North-South stationkeeping was studied in this thesis, through the use of a set of orbit inclination change maneuvers each year. Two cases for the maximum number of maneuvers to be executed were considered, with four and five maneuvers per year. The results delivered by the algorithm provide maneuver schedules which require 40 to 100 m/s of total Δv for two years of operation, with the satellite maintaining the satellite's orbital plane to within 0.1° between 84 and 96 percent of the two years being modeled.

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NOMENCLATURE

- \vec{r} = position vector of the satellite with respect to the Earth
- \vec{r}_{Sun} = position vector of the satellite with respect to the Sun
- \vec{r}_{Moon} = position vector of the satellite with respect to the Moon
- $\vec{r}_{S/E}$ = position vector of the Sun with respect to the Earth
- $\vec{r}_{M/E}$ = position vector of the Moon with respect to the Earth
- μ_E = gravitational parameter of the Earth
- μ_S = gravitational parameter of the Sun
- μ_M = gravitational parameter of the Moon
- Ω = right ascension of the ascending node
- i = inclination of the orbit

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Chapter 1

Introduction

As commercial space companies expand their operations in space, these companies and organizations must investigate new, more cost effective methods to continue the necessary operations to help make this fledgling industry profitable. One concern is to extend the operational lifespan of their commercial satellites. The objective of this thesis is to demonstrate the application of a modern multi-objective evolutionary algorithm to this problem in order to investigate an optimal stationkeeping maneuver scheduling to ensure safer and more cost effective operational lifetimes for geosynchronous satellites by minimizing both the time out of position and the total propellant consumed over the operational period of the satellite. This is done using a procedural model, which simulates the operation of a collection of satellites over the course of three years. These satellites are considered to be under the perturbing effects of the gravitational pulls of the Sun and Moon. A direct correlation between the frequency of maneuvering and both total time out of service and total fuel consumed is expected. The results of this thesis could offer insights into a new method for planning the stationkeeping maneuvers necessary for long-term operations of satellites in geosynchronous orbit.

1.1: Motivations

The focus of this thesis is the operation of satellites in geosynchronous orbit, where a variety of satellites critical to telecommunications and national defense are operated. Ensuring longer, less expensive and safer operational lifetimes for these satellites is the most direct route to decreasing the cost and extending the operational lifetime of a satellite. Doing so involves

extending the usefulness of the available fuel supply for the necessary stationkeeping maneuvers. Profitability for the satellite, and especially for a constellation of satellites, can be increased by minimizing the amount of time an individual satellite is out of service, times when it is unable to perform the functions it was designed to carry out due to maneuvering or having its antennas or other systems shut down to avoid damage.

Also of significant concern is the ever-present possibility of collisions between satellites. Fortunately, to this point, no major collisions between satellites has occurred at geosynchronous altitudes, but the immediate and long-term effects of a single collision in low-Earth orbit can be easily seen by a quick investigation of the Iridium-Cosmos collision in 2009 [7]. Knowing these effects and the limitations of ground-based tracking systems leads to some concern as orbital altitudes about the Earth continue to become more crowded.

1.2: Method

The optimal stationkeeping problem uses a model following Cowell's Method for a satellite orbiting Earth at geosynchronous altitude, with the gravitational perturbations of the Sun and Moon. This model uses a fixed step-size numeric integrator to model the motion and operation of one satellite over a two year period under the perturbing effects of the Sun and Moon. A range of stationkeeping maneuver frequencies, from one maneuver daily to one maneuver annually, is used to demonstrate the inefficiency of arbitrary maneuver scheduling. In order to optimize the scheduling of the maneuvers, a multi-objective evolutionary algorithm will be applied to the problem, which will schedule four and five maneuvers per year for the satellite. Optimal performance of the results will be determined by the amount of total Δv consumed and the percentage of the total time in operation the satellite spends with a position that lies outside of

the desired orbital plane by more than 0.1° . Since this is a demonstration, an impulsive thrust approximation is used for this model.

1.3: Thesis Overview

Chapter 2 begins with some background information on the space environment and general practices at geosynchronous altitudes, additional information on the severity and lasting effects of satellite collisions, and concludes with an overview of the history, inspiration, operation, and application of evolutionary algorithms. That will be followed in Chapter 3 by an in-depth investigation of the problem and a derivation of the equations of motion which were used for the simulation, including the assumptions applied to make the problems tractable. Also included here is an overview of the model with an explanation of the interactions with the multi-objective evolutionary algorithm. In Chapter 4 a few representative or exceptional cases, and their results, will be considered in detail. Finally, Chapter 5 contains the conclusions reached based on the data generated from the experiments carried out using the model, as well as suggestions for future work.

Chapter 2

Background

2.1: Geosynchronous Operations

The geosynchronous Earth orbit (GEO) is extremely important to the day-to-day operations of the modern world. Many key communications satellites are operated at that altitude, taking advantage of the unique opportunity to have the satellite move at the same rate the Earth rotates, thus allowing the satellite to remain fixed relative to a stationary observer. However, this popularity can become a liability.

In order to allow for more satellites to operate at GEO, a method known as co-location is used. This assigns multiple satellites to occupy the same locations in orbit, but makes these particular locations much more crowded. These highly desired orbit slots correspond strongly to the locations on the Earth with greatest population. As of January 2004, as many as 8 satellites occupied a single 2° bin at geosynchronous altitude [1]. Understandably, the more crowded an area of space is, the greater the likelihood of collisions occurring there. As a result, effective stationkeeping methods are necessary to ensure the long-term safety of satellite operations at GEO.

There are two primary forms of maneuvers used for stationkeeping at GEO. First is east-west stationkeeping [2] [3]. This type of stationkeeping is needed to ensure that a satellite will remain in its desired parking slot at altitude. Too large of an east-west drift will put a satellite at risk of colliding with another satellite. This drift is caused by the non-uniform gravitational potential of the Earth. Even in a circular, equatorial orbit, these small variations have a measurable effect on the satellite, which must be corrected for. The other type of maneuver

needed at GEO is intended for correcting the north-south drift caused primarily by the gravitational pulls of the Sun and Moon. This is a very long period drift in the inclination of the orbit, requiring approximately 27 years to complete a single half period with amplitude of nearly 15° [4].

The particular stationkeeping methods used depend on the mission being carried out. For maintaining the formation of a constellation over time, relative stationkeeping is used. This method maintains the relative positions between the satellites in the constellation, which maintains the integrity of the formation over time. This method is ideal for applications where precise relative positioning within the constellation is needed, often for applications where continuous coverage is needed by the formation, not by any particular individual satellite. The other method is known as absolute stationkeeping. For this method, a box is defined along the formation's orbital track. This box is the region in which the satellite is to operate. Maneuvers are used to keep the satellite operating within this box at all times, with new maneuvers being carried out when the boundary of the box is reached. This method is most useful in applications where coverage of particular regions of the Earth's surface is necessary, or for the operation of an individual satellite, as opposed to a constellation. The parking slot assigned to a satellite in GEO is an excellent example of a well defined operational box, within which the satellite must remain at all times.

Currently, most stationkeeping operations are controlled from stations on the ground. When the orbit of the satellite, or constellation, being operated has had its orbit perturbed from the desired orbit, maneuvers are ordered from the ground station to correct the orbit. Since the late 1990s, autonomous orbit control systems have been developed and deployed on operational satellites. These autonomous systems reduce the operation costs for a mission by allowing the satellite to schedule and carry out its day-to-day operations, thus reducing the necessary staffing for the mission, which reduces the cost of operation.

2.2: Collisions

The potential for collisions have been present since the dawn of the space age, but have increased in likelihood in the most recent decades. A variety of causes for collision events exist, both natural and manmade.

Collisions on orbit are a threat which must always be considered in planning of satellite operations. Micrometeoroids are an ever present threat, primarily at GEO, with the likelihood of such an impact increasing at certain times of the year corresponding to various meteor showers. A variety of predictive models exist for the micrometeoroid environment, which predict the flux, mass and speed distributions of particles at orbit. [5]

A dramatic illustration of the results of a collision between satellites occurred in February of 2009 when an uncontrolled satellite, Cosmos 2251, collided with the active communications satellite Iridium 33, a member of the Iridium communications satellite constellation. This collision destroyed both satellites, shattering them into many smaller pieces. The Iridium satellite's destruction left a hole in the constellation's coverage, requiring other satellites to maneuver to cover the sudden gap. The destruction of the satellites also increased the number of uncontrolled objects in orbit. This event illustrates the multiple risks posed by collision events at orbit: they jeopardize the mission of the satellite and can produce hundreds, even thousands of addition pieces of debris, increasing the chances of future collisions [6] [7] [8] [9].

This type of collision requires significant relative velocities to occur. As a result, they mainly occur in lower orbits, where a multiple orbit inclinations are used, leading to crossing orbit tracks. Great effort has been put into researching and avoiding collisions between satellites, using predictive models to determine the probabilities of future collisions [10]. At GEO, the orbit used is in the equatorial plane, and all objects are traveling in the same direction. As a result, the relative velocities between any two satellites in orbit at GEO are very small. This means that any

inter-satellite collisions at this altitude are likely to be less dramatic than those at lower altitudes, but are by no means any less concerning [11]. Satellites are very fragile systems, independent of the altitude they operate at. As a result, even a minor collision can have significant effects on the operation of a satellite. Some examples of these effects may include the misalignment of thrusters, which hampers orbit maintenance operations, and the damaging of antennas, inhibiting the communication of the satellite with the ground.

2.3: Evolutionary Algorithms

Considering the costs and risks involved in launching and operating satellites in GEO, the use of modern techniques for optimizing the operation of these systems is of significant interest.

Evolutionary algorithms are incredibly powerful tools for optimizing and investigating interesting and important engineering problems. These optimization methods have been applied to engineering problems since their development in the 1980s. Now, advances in computer hardware allow for greater numbers of computations to be carried out in a shorter period of time and allow for more truly useful results to be obtained from a particularly powerful class of the evolutionary algorithm, the multi-objective evolutionary algorithm.

2.3.1: Operation

The operation of any genetic algorithm is derived from the basic principles of Darwinian evolution, in particular the survival of the fittest. Through the use of a cost or fitness function to rate the strength of individual members of the digital population, as well as various generational changes, a population is moved through the search space toward an optimal solution for the problem at hand.

The fitness function is a user defined function. Typically, this is some mathematical expression of the form:

$$\min (\vec{y}) = f(\vec{x}) \quad (2.1)$$

where $\vec{y} \in R^n$ and $\vec{x} \in R^m$. This minimization is typically done using some restrictions, which can be placed on both the objective and search, or solution, spaces. Doing so can significantly reduce search time by limiting the feasible search space, allowing for more of the population to search the space of interest and lead to more useful results in a shorter time.

As with biological evolution, a population is gradually moved from an initial random dispersal to at least one member being optimized for survival. The movement of the population members is done through a series of operators, namely crossover, mutation, and selection. Crossover is analogous to mating where the characteristics, decision variable values, two or more population members are combined in some manner in order to produce one or more new population members. A wide variety of crossover methods exists and are employed in different algorithms, each providing unique strengths and weaknesses. Mutation varies the characteristics of individual members randomly, expanding the search of the algorithm. Selection determines, based on the fitness of the individual population members, and determines population members that move onto the next generation. As with crossover, a variety of methods exists, each with different levels of selection pressure and competition.

Single-objective problems, such as maximizing the load a beam can carry, and multi-objective problems, such as maximizing the load a beam can carry while minimizing the weight of the beam, each have different types of solutions. As a result, the operation of single-objective and multi-objective algorithms differs slightly, mainly in the stopping criteria for the algorithm. For multi-objective algorithms, the run length is governed by the number of fitness function evaluations that are desired. Single objective algorithm run times can vary, as they operate until an optimal solution is found. A generic flow chart for the operation of a genetic algorithm can be

found in Figure 2.1. First, an initial population is created with random values for each of the attributes each member carries. The attributes of these population members are then passed to a fitness function, which returns a numerical value for the fitness level of each population member for all of the objectives being considered. The fitness values are then used in selection, where the “survival of the fittest” concept is applied. Here, the most fit members of the current population are selected to be used in generating a new generation through the use of a crossover operation. These most fit members may also persist into the next generation, depending upon the algorithm being used. At the same time, the algorithm may store the most fit members in an archive to be used in later generations, but this function also depends upon the particular algorithm being used. A mutation operator is typically applied as well, which varies the attributes of each member by a small amount. With a new generation created, the algorithm checks that the stopping criteria have been met. If they have, the results are returned, and the algorithm’s operation ceases. If they have not been met, then the algorithm passes the current population to the fitness evaluation function, continuing the loop.

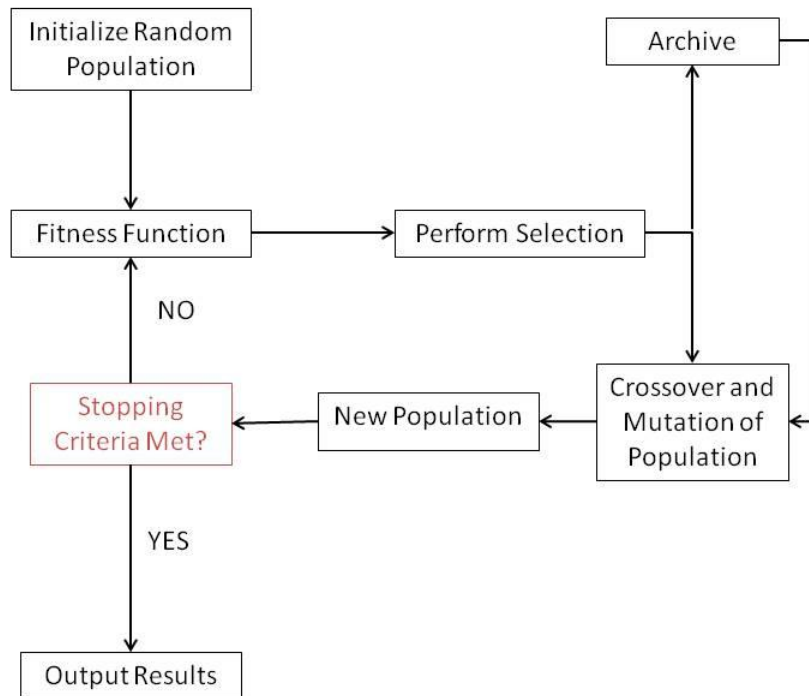


Figure 2.1: Flow Chart for a Genetic Algorithm

The use of the archive is an optional component, included in some newer algorithms. Population members with the best fitness in their generations are stored in the archive, and are often employed in operations with future generations in order to maintain greater diversity within a population, helping to prevent the algorithm from stalling.

With all of the operators, the main focus is properly combining each to provide a fast and accurate algorithm. This leads to a necessity for balance between rapid convergence, which can cause the algorithm to be extremely sensitive to local optima, and stagnation which prevents the algorithm from completely driving the population to an optimal value. These concepts are heavily discussed in the references [12] [13] [14] [15].

2.3.2: Multi-Objective Evolutionary Algorithms

The multi-objective evolutionary algorithm, or MOEA, has its own unique challenges. In order to understand these challenges and several of their solutions, a few definitions must be established. The first of these terms is non-dominance. In the context of MOEAs, a non-dominated solution cannot be outperformed by any other solution in terms of all objectives. A non-dominated set is the set of solutions which cannot be outperformed in all of the objectives by any other population member. To increase performance in any one objective, performance in at least one other objective must be sacrificed. This is the root of the trade-off concept. Illustrating the trade-offs between a collection of possible solutions makes *a posteriori* decision making possible. Another term for this collection of non-dominated solution is the Pareto optimal set.

Another term for the Pareto optimal set, often used when referring to a graphical representation of the results provided by an algorithm, is the Pareto front. These fronts are the limit on the optimization available for the problem being considered. In fact, the front separates the solution space into two distinct regions: the accessible and inaccessible regions. The points lying on the front are the non-dominated set of solutions to the problem being considered, or the representations of the optimal solutions to the problem being considered. The shape of the Pareto front is typically unique to the problem being considered. Bounds exist on the dimension of the Pareto front. The maximum dimension of the Pareto front will be one less than the dimension of the objective space.

A distinction must be made between the different conditions of Pareto optimality. As in single objective problems, a global set of optimal solutions is desired. The set of globally optimal, non-dominated, solutions is the Pareto optimal set. Typically, Pareto optimal sets can only be easily known for multi-objective test problems [16] [17] used to investigate the performance of MOEAs. For the vast majority of real-world applications, especially when exhaustive testing is

not a readily available option, the generated solution set must be termed a Pareto approximate set until some type of proof of optimality can be applied.

The quality of the results provided by an MOEA can be described in terms of three characteristics: convergence, consistency, and diversity. Convergence refers to approaching the Pareto optimal front. A fully converged solution will lie on the Pareto front. A consistent solution consists of results which all have similar levels of convergence. Obtaining a single result which is significantly more converged than the rest does not provide the trade-off information we desire from the application of an MOEA. Diversity in the solutions is also highly desired. Solutions which are clumped together offer very little insight into the trade-offs available in solving the problem. The ideal solution set displays all of these characteristics leading to a well defined Pareto optimal set, with the greatest insight into the options available to solve the problem being studied.

The stopping criterion for an MOEA is significantly different from that used by a single objective algorithm. Since a consistently, fully converged, diverse set of results is desired, the use of a measure of improvement from generation to generation is not easily defined, and is typically not sufficient. Instead, the length of time over which the algorithm is operated is used to determine when the operation should end. The parameter defining this is known as NFE, or number of function evaluations. A higher NFE total provides the MOEA with more opportunity to search the solution space, but requires more computer time and operations in order to provide the final results.

2.3.3: History

Evolutionary algorithm development began in the 1980's as a method to solve optimization problems, particularly for problems where derivative based methods were not

applicable or efficient [18]. These early algorithms sought to solve single-objective problems which had a single optimal solution.

As the broad applicability of this method became apparent, it was applied to other, more interesting engineering problems. Most engineering problems have a multitude of design criteria, such as cost, weight, and failure criteria, which must be optimized. There are often trade-offs between these objectives, where improvement in any one objective can only be obtained by sacrificing performance in another. Early work on these multi-objective problems was done using single objective algorithms being applied to each individual objective. This method proved to be clumsy, inefficient, and lacking in proofs which could be applied to demonstrate the optimality of the result.

Major changes occurred in the mid-1990's with the development of the Nondominated Sorting Genetic Algorithm, or NSGA, by Deb [19]. This algorithm provided significant performance improvements compared to the earlier MOEAS, but the algorithm used a very vague parameter known as niche radius. This parameter did not easily relate to the problem being considered but significantly affected the performance of the algorithm, mainly in terms of convergence rate. The next improvement on the algorithm was the most important, leading to the Nondominated Sorting Genetic Algorithm II, or more compactly, NSGA-II [20]. With this iteration, the use of the niche radius parameter was removed, making the results provided by the algorithm more easily understood. More recently, the ϵ -Domination Based Multi-Objective Evolutionary Algorithm, ϵ -MOEA [21], which is used in this thesis, has revisited the use of the spacing parameter in a different manner. The ϵ -dominance used in this, and other recent algorithms, serves to accelerate the convergence of the algorithm. Further developments in algorithms, such as the self-parameterizing Borg MOEA [22], have offered significant improvements in terms of speed and search fidelity.

2.3.4: Applications to Optimization Problems

Evolutionary algorithms have applications across a wide variety of engineering disciplines. Examples of these applications can be found in such varied disciplines as civil, electrical, and aerospace engineering. As varied as the disciplines are, the subjects of the applications are even more varied, ranging from water resource management to satellite constellation design.

The needs for clean, readily available water has become an increasingly important subject of interest for many communities and decision makers across the globe, driving engineers to pursue novel techniques to provide decision makers with as much accurate information as possible with as little personal bias as can be achieved. The global search methods used in MOEAs allows for all of the available options to be analyzed, with the costs and benefits of each easily returned for the consideration of decision makers. Reed et al. have used MOEAs extensively in their research and in advising policy makers in the available option in dealing with optimal allocation of water resources [23] [24].

One of the original applications of evolutionary algorithms was in the development of control laws, modifying the gains of controllers to increase the stability or performance of an engineering system. This has continued with the development of multi-objective EAs, allowing for more than a single desired trait to be investigated. Extensions of this approach have reached into the realm of fuzzy logic and machine learning, with noteworthy applications including the development of fuzzy autopilot systems, optimized using MOEAs [25].

In addition to the controls aspects of aerospace engineering, MOEAs have been applied successfully to problems in other facets of the discipline. Increasing interest in supersonic vehicles has driven the development of more safe and efficient airfoil shapes at these velocities. Such applications have led to superior designs being discovered which were previously unknown

[26]. Applications in the fields of satellite operations and constellation design and operation abound, as well. With recent successful developments and deployments of high-efficiency, low-thrust propulsion systems for use on satellites, optimal applications have been highly desired. MOEAs have been used in designing both interplanetary trajectories and orbit transfers, often focusing on the use of low-thrust propulsion systems [27] [28] [29] [30]. Satellite constellations have been employed for as long as large area coverage for communication or positioning systems has been needed. Given the national security implications of the failure of a member of the constellation, the problem of reconfiguring the constellation while maintaining both the coverage of the constellation and ensuring the longest possible operational lifetime of the constellation as a whole has been of significant interest. MOEAs proved to be incredibly well suited to investigate this problem, and offered unique insights into the problem itself [31]. Another developing technology is the use of formation flying of small satellites. These satellites offer enhanced capabilities for scientific experimentation over modern satellites, at equal or lower cost. Efficiently maneuvering these formations can pose problems, most notably due to the risks of contamination of other formation members with expelled propellant. With all of these considerations in mind, MOEAs have been applied to determining the maneuver size, direction, and coasting arc for each formation member, allowing for a safer and more cost effective operation of a formation [32].

As this technology develops, and a wider variety of engineers and scientists investigate the unique benefits provided by MOEAs, new applications will arise. The variety of these newer applications, as well as the results found, have drawn heavy attention. These trends have become so numerous and noteworthy as to spur the writing of a book on the subject. Coello Coello released a text in 2005, covering the trends in evolutionary multi-objective optimization, concerning both the developing theory and applications of these tools [33].

Chapter 3

Analysis

3.1: Overview of Problem

When orbiting the Earth, perturbing forces, of smaller magnitude than that of the gravitation of Earth, cause variations in the orbit of the satellite. These perturbations arise from a variety of sources, and vary in type and magnitude with the orbital altitude of the satellite. At geosynchronous altitude, the most significant perturbations arise from the gravitational effects of the Sun and Moon, solar radiation pressure, and zonal harmonic effects; the most significant of these are the Luni-Solar gravitational effects [34]. These perturbations cause changes in the inclination of the orbits of satellites at higher altitudes and can thus subject those satellites to other perturbations, such as zonal harmonic effects, with increased significance. The model used here incorporates the effects of the gravitational perturbations of the Sun and Moon acting on a single, hypothetical, satellite orbiting Earth at GEO.

3.2: Assumptions

A variety of assumptions and simplifications are made for this problem to allow a solution to be obtained in a reasonable amount of time. The first assumption is that the satellite is primarily orbiting the Earth, and that the Sun and Moon are applying perturbing gravitational effects to that orbit. All other forces and pressures, such as atmospheric drag, solar radiation pressure, and gravitational attraction from other planets, are being ignored. We next make the

assumption that the mass of the satellite is much less than that of the mass of the gravitational sources being considered, allowing for the following:

$$Gm_{Body} \approx \mu_{Body} \quad (3.1)$$

All maneuvers are assumed to be carried out using impulsive maneuvers. All of the velocity changes are assumed to be instantaneous, with no time-varying thrust profile. With the one minute time steps being used for this model, this approximation is quite reasonable as the burn times necessary for such small changes in velocity are likewise quite small, certainly smaller than the one minute time steps.

Following along with the previous assumption, the mass of the satellite is assumed to remain constant though the operational period being studied. In reality, whenever the thrusters are used, the satellite discharges a small amount of its overall mass. Thus, over the operational lifetimes of the satellite, the total mass can decrease by as much as 50% [34, pp. 894-5].

For this model, only the inclination regulating maneuvers are being simulated. There is no simulation of the rephasing of the satellite or for reorienting the satellite in order to carry out the maneuver. The necessary Δv for an inclination change, Δi , at any point on an orbit, is given by

$$\Delta v = 2v_0 \sin\left(\frac{\beta}{2}\right) \quad (3.2)$$

where v_0 is the magnitude of the velocity at the time the of maneuver, and β can be found using spherical trigonometry to be

$$\cos(\beta) = -\cos(i_0)\cos(i_f) + \sin(i_0)\sin(i_f)\cos(\Delta\Omega) \quad (3.3)$$

Here, i_0 is the inclination of the orbit before the maneuver, i_f is the inclination after the maneuver, and $\Delta\Omega$ is the change in right ascension of the ascending node from the initial to the

final orbit. The most efficient location on the orbit to carry out this type of maneuver is at the node crossing, where the orbit crosses the equatorial plane. At that location, no change in right ascension of the ascending node is needed, reducing the value for β , which decreases the necessary Δv . The formula for finding the necessary Δv becomes simply

$$\Delta v = 2 v_0 \sin\left(\frac{\Delta i}{2}\right) \quad (3.4)$$

3.3: Derivation of the Equations of Motion

The derivation of the equations of motion is fairly simple, following directly from Newton's Second Law of Motion,

$$\sum \vec{F} = m_{\text{satellite}} \vec{a} = \vec{F}_{G,Earth} + \vec{F}_{G,Sun} + \vec{F}_{G,Moon} \quad (3.5)$$

Through substitutions, including the assumption noted in Eq. (3.1), the governing equations of motion for this problem simplify to

$$\frac{d^2 \vec{r}^{(I)}}{dt^2} = -\frac{\mu_E \vec{r}}{r^3} - \frac{\mu_S \vec{r}_{Sun}}{r_{Sun}^3} - \frac{\mu_M \vec{r}_{Moon}}{r_{Moon}^3} \quad (3.6)$$

To simplify further, some vector relations are needed in order to reduce the computational needs. Consider Figure 3.1 below, showing the positional relationships between the Earth, Sun, Moon, and satellite.

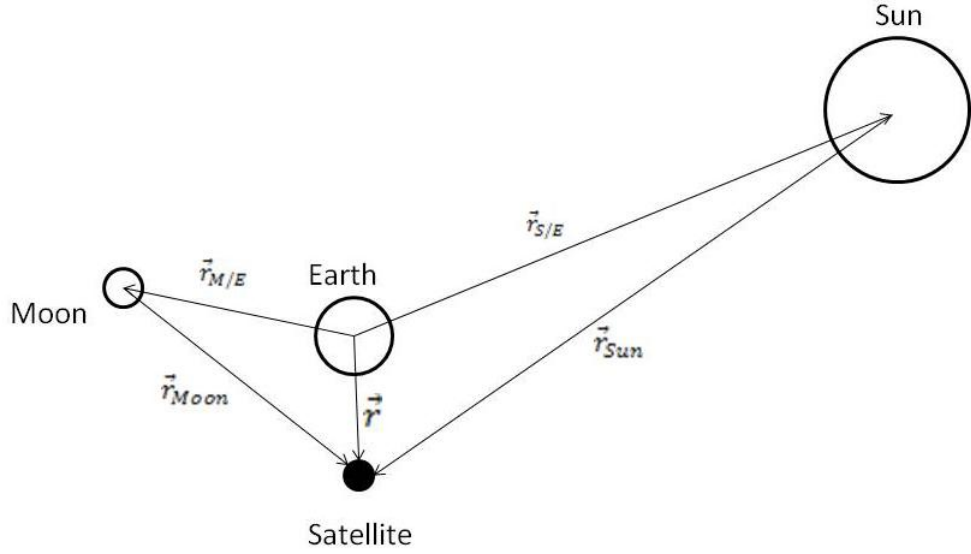


Figure 3.1: Vector diagram illustrating the geometry of the problem

Based on the vector relations, the following substitutions are employed:

$$\vec{r}_{Sun} = \vec{r} - \vec{r}_{S/E} \quad (3.7a)$$

$$\vec{r}_{Moon} = \vec{r} - \vec{r}_{M/E} \quad (3.7b)$$

As a result, Eq. (3.5) can be written as

$$\frac{d^2 \vec{r}^{(I)}}{dt^2} = -\frac{\mu_E \vec{r}}{r^3} - \frac{\mu_S (\vec{r} - \vec{r}_{S/E})}{r_{Sun}^3} - \frac{\mu_M (\vec{r} - \vec{r}_{M/E})}{r_{moon}^3} \quad (3.8)$$

The formulation of Eq. (3.8) puts the equations of motion into a convenient form for modeling. Through the use of an efficient ephemeris [35] [36], the positions $\vec{r}_{S/E}$ and $\vec{r}_{M/E}$ can be easily found at any desired time.

3.4: MOEA

This problem boils down to the optimal scheduling of a set of stationkeeping maneuvers for a satellite in GEO. The optimal timing for each maneuver, however, depends upon the previous maneuvers. These earlier maneuvers determine when the satellite will cross the desired orbital plane where a maneuver can be carried out with minimal fuel consumption. MOEAs are very well suited for solving this type of problem, making their application here quite interesting.

The model, further explained in Section 3.5, serves as the fitness evaluation function for the MOEA being used for this thesis. Due to its computational efficiency and variety of applications, the ϵ MOEA is applied to the problem. This is done through the use of the MOEA Framework, version 1.10, openly available from <http://www.moeaframework.org>. This framework provides a means of easy integration of externally defined problems with a variety of MOEAs as well as methods for analyzing the performance of the chosen algorithm for the specified problem.

The ϵ MOEA is noteworthy for applying the archive of most fit population members in the crossover and mutation operations. This allows for the algorithm to avoid stagnation in converging to the Pareto front. Figure 3.2 illustrates the operation of the ϵ MOEA, including the use of the archive members in every generation.

As with any evolutionary algorithm, an initial population with random attributes is created and evaluated for fitness. Selection is carried out using tournaments which offer the benefits of increased diversity within each generation while driving convergence. The most fit members are then saved to the archive and passed onto the crossover operation. Simultaneously, a tournament selection on the archive members is carried out. The selected archive members are also passed to the crossover operator. One regular population member is crossed with an archive member to generate new population members. This tight integration of the archive in both

selection and crossover serves to increase the diversity of the population, which provides a more thorough search of the solution space. A mutation operation is then carried out on the population members. Finally, the algorithm checks the total number of function evaluations it has completed and proceeds or closes as is appropriate.

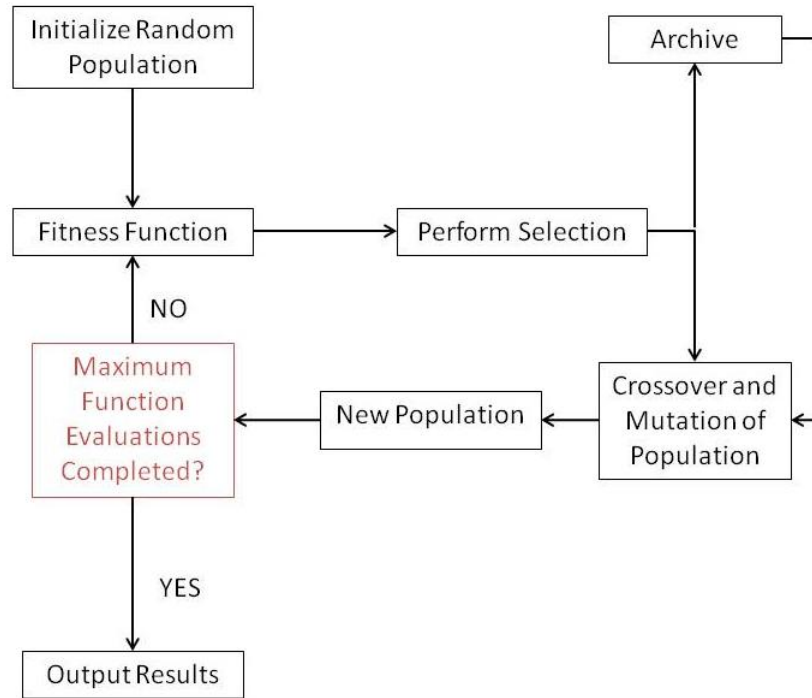


Figure 3.2: Flow Chart for the ϵ MOEA

The other noteworthy difference between the operation of ϵ MOEA and other MOEAs is the use of ϵ -dominance. A value, known as ϵ , can be defined for each objective which is being optimized. The objective space can then be broken down into a grid with cells with size ϵ_1 by ϵ_2 , as can be seen in Figure 3.3. Only one dominant solution can exist within each cell. This approach serves to increase the both diversity of the search results and the convergence rate for the algorithm. Selection is also more quickly carried out because any population members that exist in a cell dominated by any other populated cell considered fully dominated, and are therefore not considered. In Figure 3.3, if P, 1, and 2 are assumed to be population members of a particular generation, assuming that optimality requires minimizing f_1 and f_2 , selection would occur only

between points 1 and 2, as the cell they populate dominates the cell in which P populates, and thus they dominate P as well.

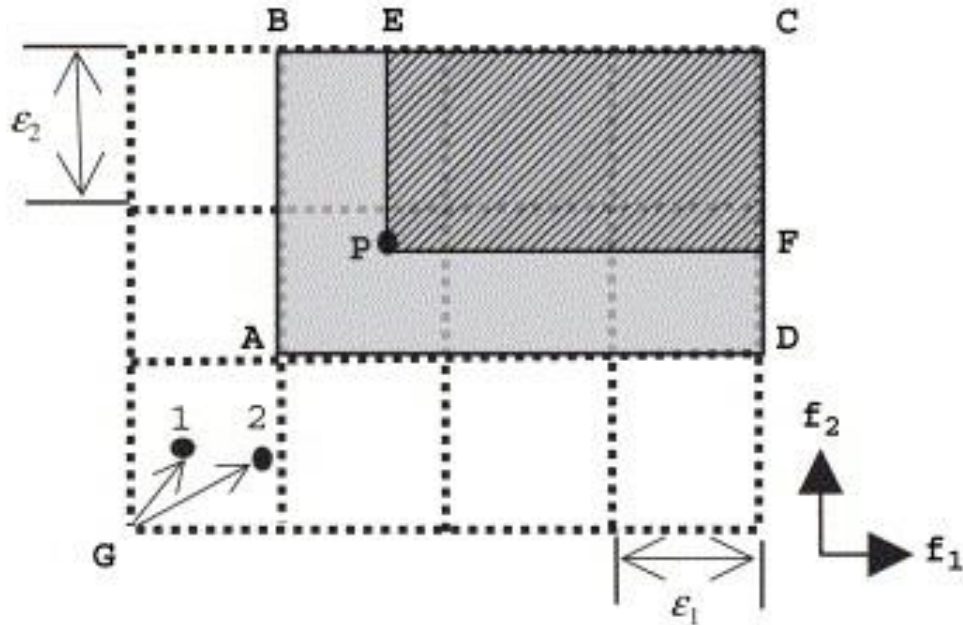


Figure 3.3: Figure Illustrating ϵ -Dominance (From Deb. Et al, 2003 [21])

For this thesis, the decision vector has length N , where N is the number of maneuvers.

Each member of this decision vector is a number τ , where

$$0 < \tau < 1 \quad (3.9)$$

This vector is passed into the model which then interprets each as the fraction of the year which must pass before carrying out each stationkeeping maneuver.

While operating, the model will compile values for two objectives: the total Δv used and the portion of the operational time being considered where the satellite has an out of desired plane position of at least 0.1° . The total Δv used provides a generic measure of propellant consumption, independent of the type of thruster used for the maneuvers. The time out of position is of interest for the effective operation of the satellite. Increased time spent out of the desired orbital plane can decrease the time where coverage is provided to a desired location. It also increases the time

where the satellite is subjected to additional perturbing forces, typically due to the non-uniform distribution of the Earth's mass.

Before returning the results to the MOEA, the time out of position is divided by the total time being investigated, so that the time out of position objective, TOP , lies between 0 and 1. Some of the total Δv values may be excessively large, and therefore unreasonable to consider. A threshold limit of 100 m/s of velocity change per maneuver, double the value suggested as necessary by Chobotov, is used to remove the results which require more velocity change. If the total Δv is greater the 100 m/s times the number of maneuvers carried out, the time out of position objective is incremented by 1. This ensures that these useless results are not carried on from generation to generation.

The algorithm is carried out using the default parameterizations for the ϵ MOEA built into the MOEA Framework. The only parameter which is varied is the maximum NFE. Three values of NFE are considered in this thesis: 1000, 5000, and 10000. The results returned for each of these different stopping criteria serve to illustrate the relationship between length of search and result quality for this problem.

3.5: Model

The model applied here employs Cowell's method [37] [38] [39] in order to numerically propagate the motion of a hypothetical satellite at geosynchronous altitude over a period of two years. In addition to the gravitational pull of the Earth, the perturbing effects of the gravitational forces applied by the Sun and Moon must also be incorporated. The position vector is initially aligned with the First Point of Aries at 00:00:00 UTC on 1 January, 2013. The satellite is initially on a circular orbit.

This model serves as the fitness function used by the ϵ MOEA. The scheduled maneuver times for each population member are passed as arguments. The objective space for the problem is two-dimensional. Through the use of the MOEA, an optimal set of maneuver schedules, all of which require a combination of the two objectives: the minimum amount of propellant used and the minimum amount of time where the satellite is displaced from the desired orbital plane by more than 0.1° . The satellite will operate under the same scheduling for both years being simulated. Values for these objectives are calculated by the model, and are returned to the MOEA, which uses the values at later stages in its operation.

The overall operation of the model and its incorporation with the MOEA is very straightforward. As decision variables, the model receives a vector of times at which the stationkeeping maneuvers should be carried out. The model then integrates the equations of motion for the satellite, while performing the maneuvers at the indicated times. After the integration is carried out, the total propellant used over the operational period of the satellite being considered, in terms of Δv , and the ratio of the total time spent with an out-of-plane displacement greater than 0.1° with the total time being simulated are determined. These two values are then passed back to the MOEA as objective values. A slight refinement of the objective values is needed. With no method for bounding the objective values, results with unreasonable objective values may be produced. In order to accommodate this, a simple correction is made to the both of the generated objective values if either one is found to exceed the reasonable limits. A flow chart for the model is shown in Figure 3.4.

Upon receiving the scheduled times for the maneuvers, the model initializes the positions of the satellite and the positions of the Sun and Moon relative to Earth. The motion of the satellite, Sun, and Moon relative to Earth are integrated over the course of a year, with the satellite performing maneuvers at the scheduled times. After the year is completed, the total Δv used and time spent out of the desired orbital plane are calculated. If the satellite has not

completed both years of simulated operation, the next year is simulated. If both years of operation are complete, the total Δv and time out of plane for both years are totaled and returned to the MOEA.

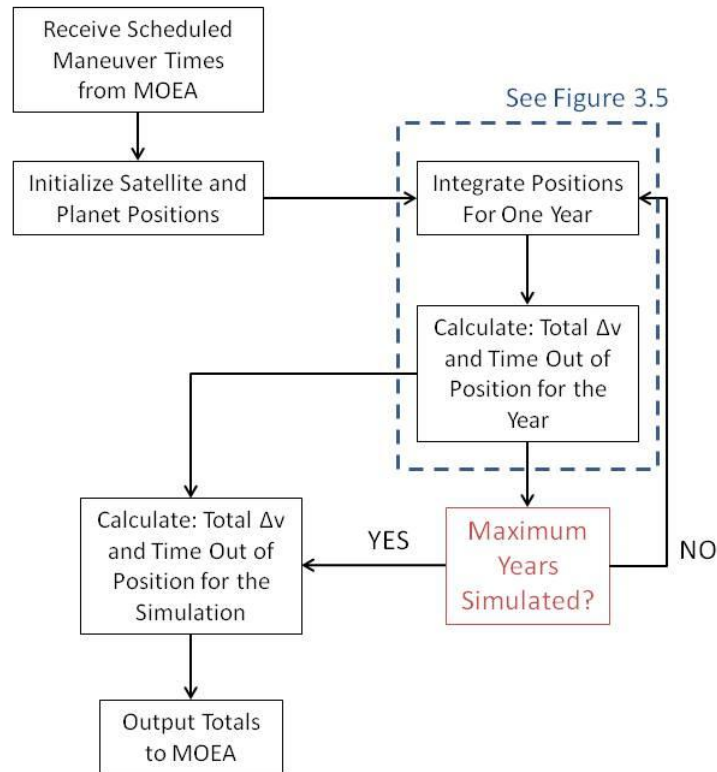


Figure 3.4: Flow Chart for Model

Integration of the equations of motion is carried out using an improved Euler method. This method was selected because it offered accuracies on the same order as the ephemeris models used, and is computationally very fast. This is desired because it allows for faster function evaluations, requiring less wall clock time in order to carry out a set number of function evaluations for an algorithm run. The inset portion of the model loop is shown in Figure 3.5.

Starting from an initial time corresponding to zero minutes into the year, the model checks to see if this is a time where a maneuver is scheduled to be carried out. If yes, then the

maneuver is carried out, the total amount of Δv used is increased by the Δv required for the maneuver. The position and velocity vectors for the satellites are corrected to reflect the maneuver was carried out. The position vectors from the Sun and Moon to the satellite are then calculated, so that the integration of the equations of motion can be carried out. The updated position vector is then used with the desired position vector to determine whether the satellite is deemed to be orbiting out of the desired orbital plane. If this is the case, then the time out of position total is incremented. The current time is then incremented, leading to a check to see if half of a day has been completed since the last time the positions of the Sun and Moon were updated. If this is the case, then updated positions are found. Another check is carried out to determine if a year of operation has been simulated. If this is the case, then this portion of the routine is exited, returning to the outer loop shown in Figure 3.4. Otherwise, the loop is re-entered and followed until the desired simulation is completed.

The desired position vector for the satellite, \vec{r}_D , is known as a function of time by

$$\vec{r}_D = r_{GEO} \cos(nt) \hat{i} + r_{GEO} \sin(nt) \hat{j} + 0 \hat{k} \quad (3.8)$$

where n is the mean motion of the satellite, found by

$$n = \sqrt{\frac{\mu_E}{r_{GEO}^3}} \quad (3.9)$$

t is the time since the investigation began, and $\hat{i}, \hat{j}, \hat{k}$ are the ECI unit vectors.

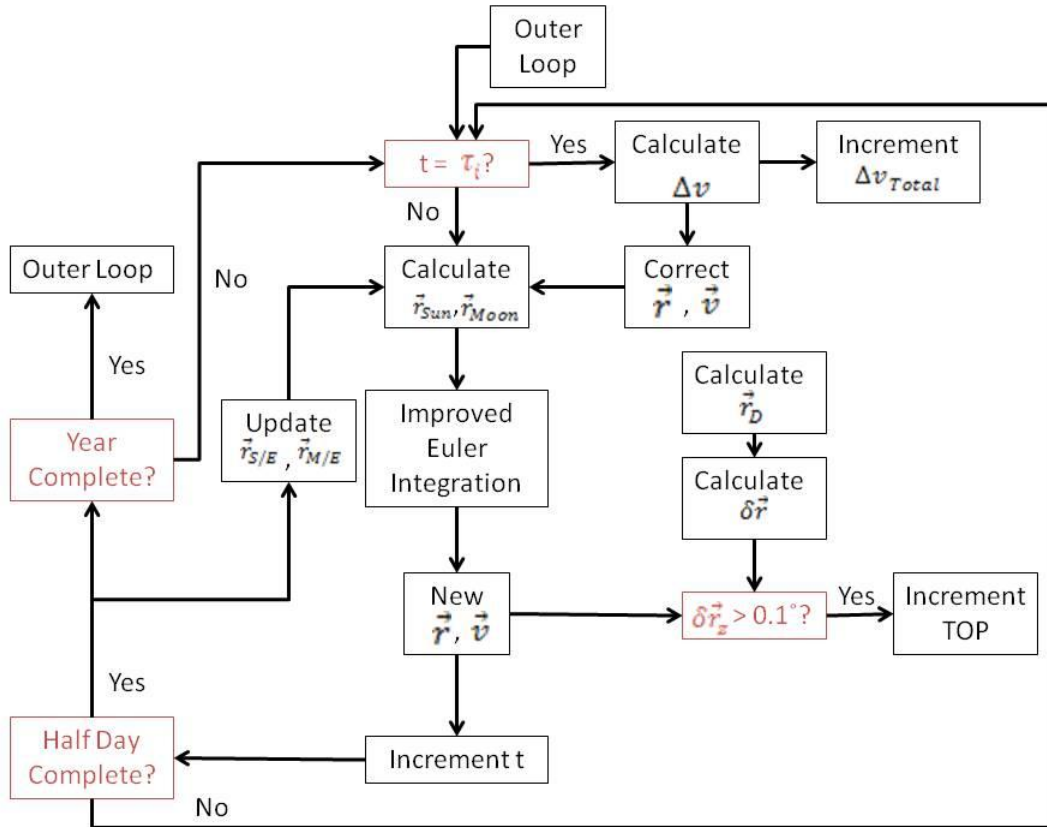


Figure 3.5: Flow Chart for Inner Loop for Model

3.6: Problem Cases

Two problem cases are used for this thesis. First is a verification case, where the only decision variable is the integer number of stationkeeping maneuvers to be carried out each year. This case is used to verify the integrity of the model, and to compare results to those already shown in literature [4]. The second, and arguably more interesting case, is the scheduling of a set number of maneuvers each year.

The verification case is used to obtain similar results to those found by Chobotov, namely that 4-5 maneuvers carried out annually can maintain the orbit of a satellite in GEO to approximately 0.1° , with each maneuver requiring about 50 m/s of Δv . The search space of this

case is very simple to navigate as it is just the set of integer numbers from 1 to 365, corresponding to the number of evenly spaced maneuvers carried out per year of operation. Given the very small, one-dimensional search space of this case, the use of a MOEA is decidedly frivolous, so the model will simply be run for each of the different values to be investigated. This verification case uses a fairly naïve approach to scheduling the maneuvers, assuming that they occur at evenly spaced intervals throughout the year. As a result, the applied velocity changes will not be minimal, as it is highly unlikely that the maneuvers will be carried out where the perturbed orbit and desired orbit intersect, which requires a change in right ascension of the ascending node for the orbit as well.

The second case has a much larger search space, with dimension equal to the number of maneuvers to be used each year. This search space is much more interesting because a wide variety of combinations of timings of maneuvers exist. The decision vector for this case will be N-dimensional, where N is the number of maneuvers to be carried out. The components of the vector are denoted τ_i , the time from the start of the year to the time when the maneuver is carried out. A slight restriction of the search space can be made by requiring that $\tau_{i+1} > \tau_i$, which reasonably requires that the scheduled maneuvers be carried out in order, and not be scheduled at the same time. Within the second case, two sub-problems are considered, scheduling four maneuvers and five maneuvers per year. These maneuver counts were chosen in order to parallel the recommendations of Chobotov.

The scheduling of the maneuvers is a problem very well suited to the application of an MOEA because there is coupling between the scheduling of the maneuvers and the occurrence of the ideal times to perform an inclination change, namely where the satellite crosses the equatorial plane. This coupling of the decision variables is easily accommodated by the search method applied by the MOEA.

Chapter 4

Results

4.1: Naïve Scheduling

The data generated from the search of the naïve scheduling space provide a baseline set of results for comparison with the results which are generated by the MOEA. The results tabulated in Table 4.1 illustrate the inefficiency of scheduling maneuvers in such a manner. The unreasonable nature of these results is shown by considering the maximum total Δv used for the operational period being simulated; the maximum value is hundreds of times larger than the Δv required to enter an interplanetary transfer trajectory. The time out of position objective, the ratio of time spent with a displacement from the desired orbital plane of at least 0.1° , also proves to have room for significant improvement.

Table 4.1: Range of Objective Values for Naïve Scheduling Method

	Minimum	Maximum
Δv (km/s)	0.0905	37393
Time Out of Position Percentage	2.80	82.05

Particular interest is placed on the sub-problems of scheduling four and five maneuvers per year. The total Δv and time out of service ration for both cases are tabulated in Table 4.2. These values provide a baseline condition which we desire the results generated by the MOEA to improve upon.

Table 4.2: Naïve Scheduling Results for Four and Five Maneuver Sub-Problems

	Maneuvers	
	4	5
Δv (km/s)	0.0905	136.0980
Time Out of Position Percentage	31.17	55.78

The Δv totals shown in Table 4.2 were produced by totaling the Δv used in 4 and 5 evenly spaced inclination change maneuvers per year for two years. The relatively large magnitudes are expected, as the likelihood that the maneuvers were carried out at or near the desired orbital plane, where the change in right ascension of the ascending node would be small, is very slim.

4.2: Scheduling With ϵ MOEA

Through the use of a MOEA, improvements in both objectives are made through adjusting the decision variables for the problem, in this case, the times at which the stationkeeping maneuvers are carried out. The solutions generated by the MOEA completely dominated the naïve solution to the problem, as desired.

4.2.1: Four Maneuver

Although the baseline results showed that the naïve scheduling of four maneuvers per year outperformed all of the other configurations in terms of propellant consumption, a time out of position of over 31% shows significant room for improvement. The schedules for the maneuvers used by the satellite, and the accompanying objective values are shown in Figure 4.1. These results were generated using different values for the maximum number of fitness function

evaluations which the MOEA is to use. The results were so dominant, in fact, that the baseline results and MOEA generated results could not be shown on the same figure while providing sufficient resolution to view the Pareto fronts. The minimum and maximum increases in performance for both of the objectives at each of the allowed NFE are displayed in Table 4.3. These increases in performance in terms of total Δv consumption range from 23.72% to 47.92%. In addition, total Δv consumption for this maneuver type ranged between 40 and 70 m/s, reduced from 90 m/s. Even more noteworthy are the improvements in the percentage of time out of position. Improvements here ranged from 50.67% to 85.54%. The performance increases are relative to the baseline results from Table 4.2.

Table 4.3: Increases in Performance with Respect to the Baseline Results for Four Maneuvers

NFE	Percent Change from Baseline			
	Δv		Time Out of Position	
	Minimum	Maximum	Minimum	Maximum
1000	26.80	40.53	50.67	80.68
5000	23.72	35.18	57.91	85.54
10000	29.14	47.92	56.47	85.27

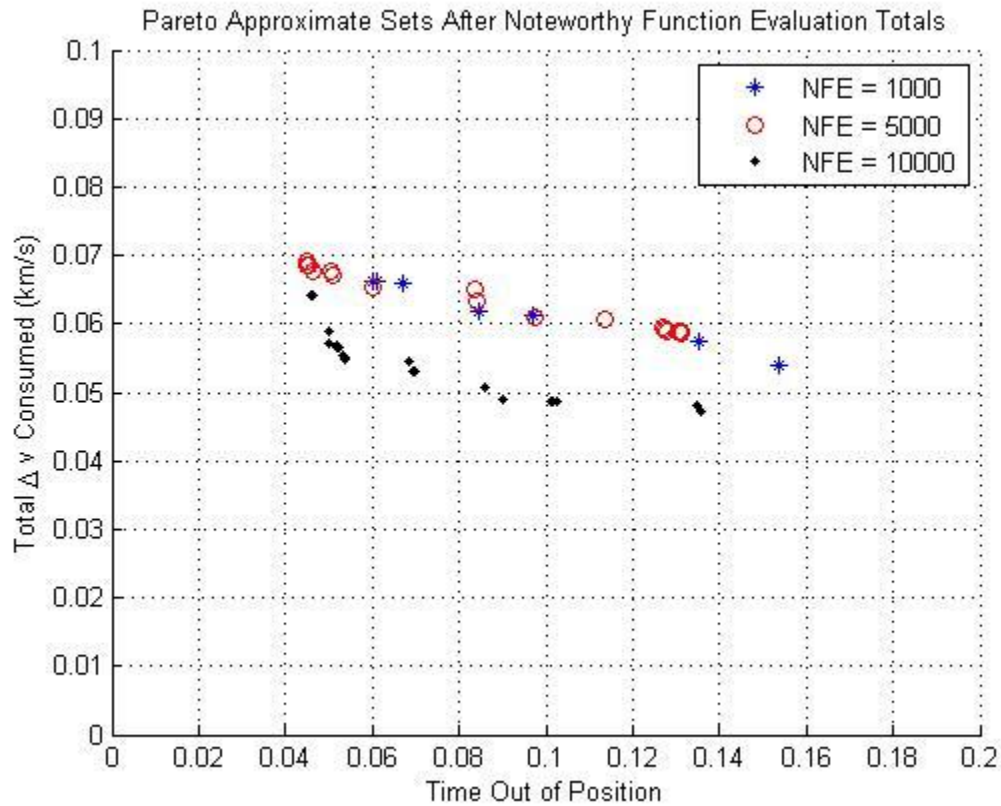


Figure 4.1: Development of the Pareto Front with varying Function Evaluations for the four maneuver sub-problem

4.2.2: Five Maneuver

The five decision variable problem produced similar results to the four variable problem. Great room for improvement existed for this problem, with a baseline required Δv of over 136 km/s and over 55% time out of position. The improvements for this problem resulted in maneuver schedules providing similar Δv requirements to those for the four maneuver case, but provided even better results in terms of the time out of position for the satellite. The tabulated improvements are quite large, but are not unreasonable when considering the poor performance from the naïve search. The performance increases are relative to the baseline results from Table 4.2.

Table 4.4: Increases in Performance with Respect to the Baseline Results for Five Maneuvers

NFE	Percent Change from Baseline			
	Δv		Time Out of Position	
	Minimum	Maximum	Minimum	Maximum
1000	99.9486	99.9550	99.8391	99.93873
5000	99.9536	99.9610	99.7418	99.9732
10000	99.9548	99.9605	99.9587	99.9891

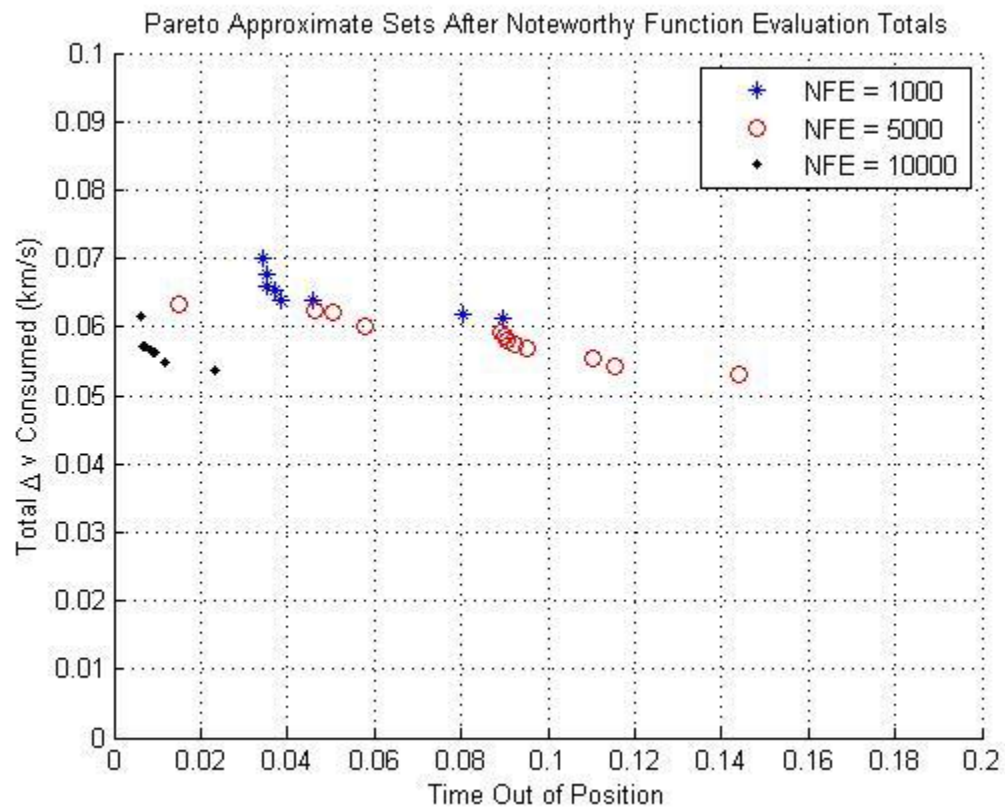


Figure 4.2: Development of the Pareto Front with varying Function Evaluations for the five maneuver sub-problem

More noteworthy than the relative improvements are the final ranges in objectives returned by the MOEA. All of the optimal maneuver schedules require total Δv less than 100 m/s, and ensure the satellite operates outside of the desired orbital plane by no more than 16% of the time it is in orbit.

4.3: Analysis of Results

Significant improvements over the naïve scheduling method are seen for both sub-problems. The maneuver schedules developed using the MOEA vastly outperform the naïve scheduling method in terms of both the total propellant consumed during the course of operation, and the time the satellite is displaced from its desired orbital plane.

The large search space of coupled decision variables displays noteworthy dependence on the number of function evaluations used by the algorithm. Significant changes in the location of the Pareto front developed after 10000 function evaluations can be seen in both the four and five maneuver problems, as shown in Figure 4.1 and Figure 4.2. The most significant changes are seen when comparing the Pareto fronts after 5000 NFE and 10000 NFE. The 10000 NFE Pareto front dominates the front after 5000, as performance in both objectives is improved for both sub-problems. The complete domination of the naïve solutions used as a basis for comparison supports the application of this method for developing maneuver schedules for satellites.

The Pareto approximate set for the four maneuver problem exhibits better diversity than the five maneuver results. This provides a more well-defined Pareto front, which helps to better illustrate the available trade-offs. The results for both sub-problems showed excellent consistency, with no apparent outliers existing in the non-dominated sets.

Overall, the significant improvements in both objectives suggests that the use of such a method for the scheduling of stationkeeping maneuvers is a viable option. The relationship

between the NFE and level of convergence is not unexpected. With larger maximum NFE comes increased time where the algorithm can search the solution space for optimal configurations. Run serially on a 3.42 GHz AMD processor, the 10000 NFE runs required approximately three hours each to complete. Larger runs would reasonably take longer. However, the use of parallelization techniques, namely carrying out the fitness evaluations on separate processors would significantly reduce the time to perform the same search.

This approach offers unique advantages over more conventional, human-centric, approaches, where the maneuvers are scheduled individually, often as they become necessary. Through the use of an MOEA, any number of maneuvers can be scheduled prior to the launch of a satellite, assuming that sufficiently accurate ephemeris information is available. Additional constraints can also be placed upon the solution space, such as requiring a particular maneuver to be carried out at a specified time. Multiple schedules can be developed in a matter of hours, whereas developing similar results conventionally could require days. Solving in parallel offers additional opportunity for scalability, as increasing the number of processors in a computer system is typically easier, faster, and cheaper to perform when compared to adding additional staff.

Chapter 5

Conclusions and Future Work

This thesis has shown that scheduling stationkeeping maneuvers using an MOEA is a viable option that provides significant decreases in the time the satellite spends out of the desired orbital plane and the required Δv used in stationkeeping when compared to scheduling maneuvers at evenly spaced maneuvers every year. In practice, the results returned would provide satellite operators a variety of options for maneuver schedules, with each option providing unique benefits in terms of propellant consumption or time out of position. This variety would allow for the mission to be more heavily analyzed before launch, which in turn can allow for more accurate budgets for propellant consumption or insight into needs to investigate other possible issues with the long-term operation of the satellite, such as coverage concerns.

Additional applications of this method exist and warrant further investigation. Expansions on the model described here are many in number. One notable possible extension is the inclusion of east-west stationkeeping maneuvers, in addition to the north-south maneuvers used in this thesis. Such a study could provide improved scheduling guidelines or practices, improving the effectiveness of satellite operations. Such a study could be further expanded upon by applying this method to an actual mission, and comparing the MOEA generated results to the actual operation of the satellite.

Some additional possible options here include using higher accuracy ephemeris data, or accounting for other perturbations, such as solar radiation pressure. A variety of other thrust models could be considered, including low-thrust thrusters, constant magnitude or even ramp thrust profiles. There is also no reason this method could not be applied to other orbits, at lower

altitudes or other inclinations. Changing these altitudes and inclinations introduces other possible perturbations, such as atmospheric drag and oblateness effects.

It should be noted, once again, that a word of caution is needed concerning the use of multi-objective evolutionary algorithms. Algorithm operation is carried out for an established number of fitness function evaluations. The time required to carry out an individual run for a real world cost function is typically much greater than the time needed for the evolutionary algorithm to be carried out. As a result, with greater wall-clock time needed for the function to be evaluated, fewer function evaluations can be carried out in a set length of time, reducing the amount of the search space which can be covered. So, in practice, although a more accurate model is always desired, the time necessary to evaluate this model may seem restrictive. Currently, the most effective approach to dealing with this problem is distributing the operations out over multiple processors, thus increasing the amount of evaluations which can be carried out in a set amount of wall time. The availability of the necessary resources for such an approach should always be considered before being applied, as a significant investment of time must be given in configuring the problem and evolutionary algorithm to operate on multiple processors.

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