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**METHODS FOR IDENTIFYING COST-EFFICIENT TRACKING SOLUTIONS FOR
INTERPLANETARY SPACECRAFT**

A Thesis in

Aerospace Engineering

by

John P. Muncks

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The thesis of John P. Muncks was reviewed and approved* by the following:

David B. Spencer
Professor of Aerospace Engineering
Thesis Advisor

Robert G. Melton
Professor of Aerospace Engineering

George A. Lesieutre
Professor of Aerospace Engineering
Head of the Department of Aerospace Engineering

*Signatures are on file in the Graduate School

ABSTRACT

In this thesis, a process to identify cost-efficient tracking solutions for interplanetary missions is presented. The process is flexible enough to be used in a wide variety of cases with different constraints. Mission designers will be able to use this process to not only identify the most cost-efficient tracking methods for any particular mission, but also to learn about the characteristics of a cost-efficient solution.

In the process, complete tracking solutions are generated, tested, and compared. Tracking solutions combine a tracking schedule, defining when and for how long a spacecraft is tracked, and an antenna configuration, defining which antennas are used. A representation of tracking schedules is used that defines all schedules as a function of the number of tracking intervals and the grouping of these intervals into sets. This representation allowed a wide range of potential solutions to be tested while also being simple enough to offer easy identification of trends in the final data.

The process presented in this thesis consists of selecting a range of the variables defining each tracking solution, identifying constraints that limit the choices for these variables, and testing and comparing the efficiency of each of the available solutions. The tracking efficiency, a variable defined in this thesis, is used to compare the solutions. It is a measure of how well the observations of a tracking solution are converted to measurable decreases in the uncertainty in the estimate of the spacecraft's state.

In order to demonstrate the results of this process, a representative lunar trajectory is examined. This demonstration includes the selection of the range of independent variables to test, the specific parameters of the trajectory to be tested, and the simulator that will be used. Several cases, each with different constraints, will be considered in the demonstration.

The results of the process allow for quick identification of the most cost-efficient solution. They also show trends in the independent variables that show the sensitivity of cost-efficiency to each of the variables. These trends allow the mission designer to understand the trade-offs in their selection of the variables. By identifying these trends and patterns, more customizable searches can be used to obtain more specific results. The process created an end product that met all of the research goals.

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Chapter 1

Introduction

This thesis presents methods for identifying tracking solutions for interplanetary spacecraft that offer sufficient knowledge of the spacecraft's state over the course of the mission while minimizing the amount of observation time, and therefore the mission operations cost. To demonstrate these methods, the process of identifying a low-cost tracking solution is shown for a spacecraft on a representative lunar landing trajectory under several different conditions. These cases include different constraints that limit the options the mission designer can choose from when selecting a tracking solution. In each case, solutions consisting of several different tracking schedules and antenna configurations are compared. Simulated ground station observations of the satellite for each of these potential solutions are generated and subsequently processed through an extended Kalman filter. The performances of each tracking solution are compared using a value defined as "tracking efficiency," which is a measure of the efficiency of utilizing spacecraft observations to minimize the uncertainty in estimates of the spacecraft state.

Over the last decade and continuing through today, the private space industry has experienced substantial growth. New companies are frequently entering the market that promise to provide access to space that is more efficient, more frequent, and more cost-effective than the typical operations of the past. These emerging companies form a collective group named "NewSpace".

So far, none of these NewSpace companies have ventured further than geostationary orbit. However, several have plans to conduct missions to the Moon, asteroids, and even Mars. The companies targeting these locations plan to conduct a wide range of activities there, ranging from resource extraction to permanent human settlements.

For every one of these companies, the success of their missions will rely on their ability to communicate with their spacecraft from Earth. As these companies, and future companies that are yet to be established, launch their spacecraft and execute their missions, the number of objects in interplanetary space will grow. However, the number of ground stations that currently exist to support interplanetary spacecraft are limited, and many NewSpace companies will be unwilling to fund construction of their own ground stations. For these companies to succeed, they will need to rent time on existing deep space networks that are already in high demand by planetary science missions launched by national space agencies. In order to lower tracking costs, NewSpace companies will need to develop cost-effective tracking solutions that minimize the amount of tracking time while still offering sufficient knowledge of the spacecraft's state.

Terminology

An important point should be made here about the terminology used in this thesis. When monitoring Earth orbiting spacecraft with ground stations, there are separate communications passes (where the spacecraft relays data back to Earth) and tracking passes (where, in many circumstances, a radar signal is projected from the ground station and reflected off the spacecraft, which provides information about the spacecraft's position and velocity). However, when monitoring interplanetary spacecraft, there are not two distinct types of passes. Instead, tracking data is derived from the properties of the spacecraft's communication signal. Whenever an interplanetary spacecraft is communicating with the ground, it is also being tracked.

Throughout the thesis, tracking of interplanetary spacecraft is discussed, as are numerous terms such as "tracking schedules," "tracking solutions," and "tracking intervals". It must be emphasized that these terms, in reality, describe when and how to use a ground station to communicate with the spacecraft, and tracking data is derived from each communication.

Dedicated tracking schedules exist for Earth orbiters, but they do not exist for interplanetary spacecraft. Therefore, a mission designer will use the results of this analysis to derive a rough order of magnitude estimate of the communications schedule that offers the most cost-efficient amount of tracking data.

Motivation

The error in the knowledge of the spacecraft's state is inversely correlated with the number of measurements taken (except in cases where the spacecraft and/or sensors behave unexpectedly, which is described later). Ideally, operators would prefer constant observation of their craft, as it would offer them extremely accurate knowledge of the spacecraft's state when the spacecraft is broadcasting. However, there are two major reasons why taking constant measurements of a single spacecraft are undesirable for both the spacecraft operator and for the ground station's management.

The first is mission cost. When a group uses a ground station to communicate with their spacecraft, it costs money to rent tracking time on that station (the exact cost is subject to change over time, and a specific value is not used in this thesis). For the Deep Space Network[†], managed by NASA and one of only two networks that are able to continually monitor interplanetary spacecraft as the Earth rotates (the other being the European Space Tracking Network[‡], ESTRACK, managed by the European Space Agency), the costs of reserving time can be very high. For many of the NewSpace companies coming to the market looking to execute interplanetary missions, this cost of communicating with their spacecraft may be prohibitive if a tracking schedule is not efficiently designed. To lower mission cost, it will be in each company's

[†] <http://deepspace.jpl.nasa.gov/>; accessed 10/4/14

[‡] http://www.esa.int/Our_Activities/Operations/Estrack_tracking_stations; accessed 11/12/14

best interests to minimize the time they are using ground station antennas, especially if it is a specialized and expensive network like the DSN.

The second is mission mix. NASA, being the owner of the DSN, is a science and exploration organization and will drive to support as many missions as they can in order to collect the most scientific data and support as many exploration activities as possible. In order to maximize the number of missions being supported, the DSN will seek to minimize the amount of time each individual mission uses the network.

Together, these two factors drive to minimize the amount of time the DSN is used to track a single spacecraft. Yet, as stated earlier, the desire to have extremely accurate knowledge about the state of a spacecraft drives to maximize the number of observations of a spacecraft. Therefore, there is a trade-off between spacecraft state knowledge and mission cost/mission diversity, which spurs the interest in understanding methods for generating cost-efficient tracking solutions with sufficient performance.

Research Goals

The goal of this thesis is to show a process to create graphs that will allow a mission designer to quickly identify ways to achieve a cost-efficient tracking schedule. First, the end product should allow for identification of rough order of magnitude estimates of the parameters defining the most cost-efficient tracking solution. Second, the end product should show the sensitivity of the cost-efficiency of a solution to each parameter, to demonstrate to the mission designer which variables are most important. Third, the end product should allow the mission designer to estimate the robustness of the selected solution to changes in the constrained variables. Finally, the end product should display a wide enough range of solutions that patterns can be identified to guide a future search. The process in this thesis is not meant to be the final

step in selecting a tracking solution, and should be followed with more customizable searches informed by the process' results.

Analysis Overview

When a mission designer is choosing how to communicate with a spacecraft, there are two pieces that they must determine: the schedule that defines when communication is turned on, and the antenna configuration that defines which ground stations are used for receiving the communications signal. The combination of these two parts (the usage of a particular schedule with a particular antenna configuration) create what is defined in this thesis as a "tracking solution" for the mission. This thesis assumes there are three variables that the mission designer can define in their selection of a tracking solution: the total number of tracking intervals in the schedule, the grouping of these intervals in the schedule (which indirectly defines the lengths of the gaps in coverage), and the antenna configuration used (the term "antenna configuration" includes information about the number, size, and location of the selected antennas).

However, there are situations where the mission designer will be limited in their selection of these variables. The constraints limiting a mission designer include both hard constraints (when a variable must be equal to exactly one value) and soft constraints (when a variable can be equal to a range of values). If, for example, several antennas have already been reserved for another mission, and there are only a few unreserved antennas, this is a soft constraint, as the number of antennas used in the antenna configuration are, at maximum, all of the available antennas.

This thesis presents methods for selecting cost-efficient spacecraft tracking solutions under a range of different constraints. These methods are demonstrated by conducting the analysis required for a spacecraft on a hypothetical lunar landing mission and presenting the

results. Exact results are presented as examples and are applicable to the mission used in the analysis. The process, however, is applicable to any interplanetary mission.

In the analysis, seven cases are considered, each with different constraints. For every case, several tracking schedules are tested in combination with several antenna configurations to determine which tracking solution offers the best performance. Performance is calculated as a function of both the average uncertainty in the estimate of the spacecraft state over the length of the mission and the number of observations taken of the spacecraft. These two values together define a solution's "tracking efficiency," which is a measure of the effectiveness of converting measurements to decreases in uncertainty in the estimate of the spacecraft state.

Thesis Overview

Chapter 2 presents a brief background of the topics relevant to the analysis. First, the tracking methods used for interplanetary spacecraft are described. It covers the theory and equations behind the time delay measurements used to calculate range and the Doppler tracking used for range rate measurements. The second section of the chapter covers the characteristics of the Deep Space Network. Antenna size options at each of the three DSN locations and a brief discussion of the relative costs of using each dish size for tracking are discussed. The third section of the chapter discusses the Extended Kalman Filter, the process it follows and the equations it uses.

In Chapter 3, the analysis steps are outlined. First, a more detailed view of the proposed process is described. Second, the parameters and the design of the sample orbit to be used for the demonstration of the analysis are shown. Next, the set-up and functionality of Orbit

Determination Tool Kit (ODTK)[§], the program that generates and filters the simulated tracking data, are covered. After that, the generation of the custom tracking schedules are discussed, including the definition and representation of the two variables that are used to define each schedule (total number of tracking intervals and number of intervals per interval set). Next, the antenna configurations to be tested are presented. Finally, the tracking efficiency value used to compare the performance of each tracking solution is described, including the equations used to calculate it.

In Chapter 4, the methods for identifying cost-effective tracking solutions is shown for several scenarios using a representative lunar landing trajectory. Seven different cases are considered, each with different constraints. Mission designers will be able to use the methods presented in this thesis to generate rough order of magnitude estimates of cost-efficient tracking schedules that meet the constraints of the mission.

Finally, in Chapter 5, conclusions are stated. Opportunities for future work and additional analysis are also presented, in case any researchers wish to build on the methods shown here.

[§] <http://www.agi.com/products/odtk/>; accessed 8/26/14

Chapter 2

Background

In this chapter, three topics will be covered that constitute the background of the analysis in this thesis. First, the tracking of interplanetary spacecraft will be discussed. This will include the equations used by analysts to calculate the time delay and Doppler shift of the spacecraft's communication signal, from which range and range rate is determined. Second, the Deep Space Network, one of the only two networks in the world that can provide uninterrupted communications with interplanetary spacecraft, will be shown. Antenna sizes, locations, and a rough order of magnitude cost model for time rented on the network will be discussed. Third, the Extended Kalman Filter will be presented. This filter is used commonly in orbit determination methods to process noisy tracking data and determine a spacecraft's true state.

Tracking of Interplanetary Spacecraft

In order to determine the state (position and velocity) of a spacecraft, the range and range rate of the craft relative to the ground station is extracted from the spacecraft's communications signal. These observed values, when used in combination with the expected values calculated with the system's equations of motion, provide sufficient information for the orbit determination program to determine the spacecraft's position and velocity vectors with respect to an Earth-centered coordinate frame. Analysts are able to determine the range and range rate by looking at two properties of the spacecraft's communications signal: the time delay and the Doppler shift.

Using Time Delay for Range Determination

By measuring the time delay between when a communications signal is transmitted to the spacecraft and when the return signal is received by the ground station, the range between the spacecraft and the ground station can be computed. Most of this delay comes from the signal taking time to travel through space at the speed of light. There is also small contributions due to both atmospheric attenuation and the processing of the signal on board the spacecraft, as the signal takes time to be received, processed, amplified, and retransmitted. However, this processing time can be measured before launch and then subtracted from the measured time delay.

Using the time delay, the range can be calculated using Equation (2.1), where r is the range between the transmitter and the receiver, c is the speed of light, τ is the time between the signal transmission and signal reception, and t_p is the time delay due to the signal processing on-board the spacecraft

$$r = c * \left(\frac{\tau - t_p}{2} \right) \quad (2.1)$$

In order to derive a value for τ from the waves of the communications signals, the Deep Space Network measures the phase change between the phase of the transmitter's clock when a signal is transmitted and the phase of the receiver's clock when the return signal is received. The calculation is shown in Equation (2.2)¹, where t_T is the time of signal transmission, t_R is the time of signal receipt, $\psi_T(t_T)$ is the phase of the transmitter's range clock at the time of signal transmission. $\psi_R(t_R)$ is the phase of the receiver's range clock at the time of signal receipt, $d\psi/dt$ is the rate of change of the signal's phase, and τ is the time delay between signal transmission and receipt

$$\psi_T(t_T) - \psi_R(t_R) = \int_{t_R - \tau}^{t_T} \frac{d\psi}{dt} dt \quad (2.2)$$

Upon closer inspection of Equation (2.2), it is seen that an unambiguous value of τ cannot be found. Since a signal's phase repeats periodically, the equation has infinite possible solutions for τ . However, when Equation (2.2) is solved alongside the predicted state of the spacecraft based on past observations, an unambiguous value for range can be found. This process of calculating range based on past values of range is called sequential ranging.

Using Doppler Shift for Range Rate Determination

By measuring the Doppler shift of the spacecraft's communication signal, the range rate between the spacecraft and the ground station can be found. The Doppler shift is a measurable effect caused by the motion of a transmitter relative to a receiver. For any object that generates a frequency, whether it is a sound wave or a light wave, the motion of the transmitter relative to the receiver causes the frequency to change slightly. If the transmitter and receiver are moving closer together, the measured frequency appears higher than the transmitted frequency. If the two are moving farther apart, the measured frequency appears lower.

Doppler measurements for spacecraft tracking can be taken by three different methods: one-way, two-way and three-way². One-way Doppler measurements are when the spacecraft transmits a signal at a known frequency and the ground station measures that signal to determine velocity. In two-way and three-way Doppler measurements, the signal originates from the ground station, is received and amplified by the spacecraft and retransmitted back to Earth, and the return signal is received by a ground station. In two-way Doppler, the same ground station that sends the uplink signal also receives the downlink signal. In three-way Doppler, the uplink station and downlink station differ. These three methods are shown graphically in Figure 2.1. Note that the

graphic merely shows the communication paths, and does not show the actual change in frequency due to the Doppler Effect.

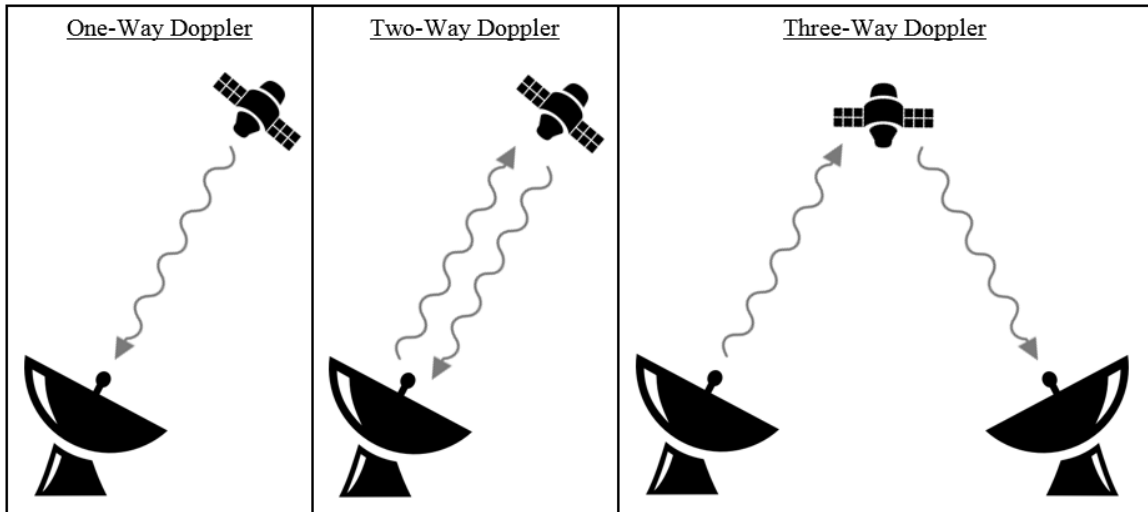


Figure 2.1: Graphical Representation of the Three Methods of Doppler Tracking

Regardless of which of the above methods is used, the core of the Doppler shift calculation is the same. The transmitted frequency is known. By comparing the received frequency with the known transmitted frequency, the radial velocity between the ground station and the spacecraft can be calculated using Equation (2.3), where f_0 is the transmitted frequency, f_r is the received frequency, Δv is the radial velocity of the transmitter with respect to the receiver, and c is the speed of light

$$f_r - f_0 = \frac{\Delta v}{c} f_0 \quad (2.3)$$

Deep Space Network

All American interplanetary spacecraft launched since 1961 have used the Deep Space Network (DSN), managed by NASA, for tracking and communication³. The DSN consists of three sites spaced evenly around the world, at intervals of approximately 120 degrees longitude. This arrangement allows the network to maintain uninterrupted communication with interplanetary spacecraft as the Earth rotates. The three sites are located in Canberra, Australia, Madrid, Spain, and Goldstone, California, as shown in Figure 2.2.

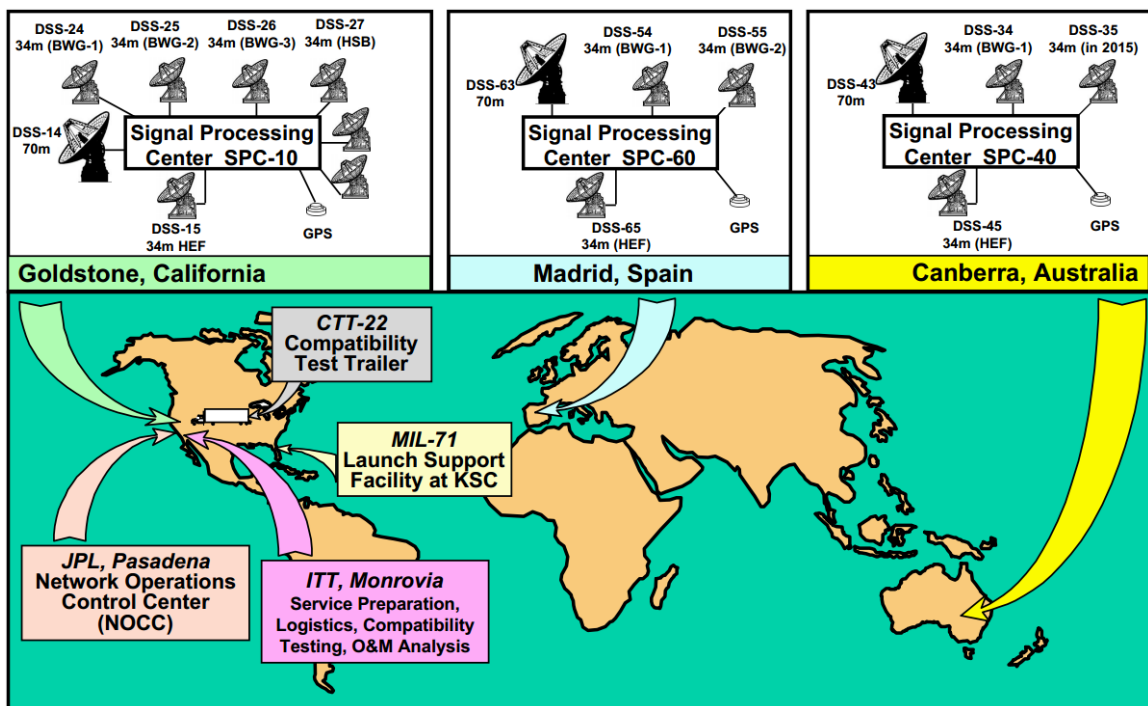


Figure 2.2: Deep Space Network Complexes⁴

Each of these sites has at least two 34-meter dishes and one 70-meter dish. The 34-meter dishes are of three different types: High-Speed Beam Waveguide (HSB), Beam Waveguide (BWG), and High-Efficiency (HEF). These types describe how each antenna takes measurements and is a function of how the antenna was constructed and the signal processing equipment it

contains. The specific way each of these antenna types operates are not covered in this thesis.

However, it is important to know that there are three types of 34-meter dishes in the DSN, as the costing models used in this thesis are dependent on the type of antenna, both size and type. A list of the antennas that compose the Deep Space Network, including their sizes and types, is provided in Table 2.1.

Table 2.1: Antennas of the Deep Space Network^{5,}**

| Station | Name | Size | Type |
|-----------------------|--------|----------|------|
| Goldstone, California | DSS-13 | 34-meter | BWG |
| | DSS-14 | 70-meter | |
| | DSS-15 | 34-meter | HEF |
| | DSS-24 | 34-meter | BWG |
| | DSS-25 | 34-meter | BWG |
| | DSS-26 | 34-meter | BWG |
| | DSS-27 | 34-meter | HSB |
| | DSS-28 | 34-meter | HSB |
| Canberra, Australia | DSS-34 | 34-meter | BWG |
| | DSS-35 | 34-meter | BWG |
| | DSS-43 | 70-meter | |
| | DSS-45 | 34-meter | HEF |
| Madrid, Spain | DSS-54 | 34-meter | BWG |
| | DSS-55 | 34-meter | BWG |
| | DSS-63 | 70-meter | |
| | DSS-65 | 34-meter | HEF |

When missions want to use the DSN communicating with their spacecraft, they are required to pay for usage time. As the exact cost of DSN support changes over time, a specific value is not used in this thesis. However, NASA has developed an algorithm that estimates the cost of using the DSN⁶, and though exact values aren't used, the weighting scheme the algorithm implements to calculate the cost as a function of antenna size is used in the performance calculation of each solution. The algorithm is shown in Equation (2.4), where AF is the weighted aperture fee per hour of use (a set value defined by NASA), R_B is the contact

^{**} Active antennas as of November 2014

dependent hourly rate, A_W is the aperture weighting, and F_C is the number of station contacts per calendar week

$$AF = R_B[A_W(0.9 + F_C/10)] \quad (2.4)$$

The aperture weighting term is designed to incentivize use of the smaller antennas. The value of the aperture weighting term, A_W , is 0.80 for 34-meter HSB stations, 1.00 for 34-meter BWG and HEF stations, and 4.00 for 70-meter stations⁶. This means time on the 70-meter dishes costs at least four times as much as time on any of the 34-meter dishes. This weighting scheme is used for calculation of the tracking efficiency, further described at the end of this chapter. When calculating the tracking efficiency of each tracking solution, the total number of observations taken is a weighted value, where observations from 70-meter dishes contribute four times as much as observations from 34-meter dishes.

Extended Kalman Filter

When communications signals are received by ground stations, the signals are inherently noisy. This noise can come from a variety of sources, including atmospheric effects and electrical noise. In order to process the noise out of the communications signals and develop spacecraft tracking data to determine the true state of the spacecraft, a Kalman filter is often used⁷. The Kalman filter was designed by Rudolf Kalman in the 1950's to process noisy measurement data for a system with known equations of motion⁸. It can be characterized as an adaptive least-squares sequential filter.

The Kalman filter, in its most basic form, works by comparing the observed/measured values from sensors and tracking stations to expected values calculated from the equations of motion of the spacecraft. However, it contains several features that make its estimates more

accurate over time than a typical least squares filter. Arguably the most significant feature is that it is adaptive. The more measurements the filter processes, the more accurate its estimates become. With every measurement, the filter calculates the deviation between what the equations of motions indicate the measured value should be and what the actual measurement is. This deviation is also known as the residual. If there is a large residual in the measurement, that means that either 1) the measurement is in error, possibly because of a faulty sensor or a lot of noise, or 2) the equations of motion or one of the constants used in the equations of motion defining the system is incorrect. The Kalman filter is able to store this inconsistency in a matrix that is updated with each new measurement. This matrix, called the covariance matrix, is stored for the length of the mission and acts as a sort of “memory” for the filter. It stores the calculated uncertainty of the filter’s estimate of the state. If a residual is measured consistently, the values in the covariance matrix grow, indicating there is a detected error in that value, and the filter adapts to a solution that better fits the data it has received.

The extended Kalman filter operates in a similar way to the standard Kalman filter but it is better suited for non-linear problems⁹. The equations of motion of an orbit are all non-linear, so an extended Kalman filter should offer better results than a standard Kalman filter. The steps and equations in the Extended Kalman filter are shown in Figure 2.3, a flow chart sourced from Tapley, Schutz and Born in Ref. 9.

In the flowchart, several variables are used. \mathbf{X} is a vector representing the spacecraft’s state (position and velocity) and $\dot{\mathbf{X}}(t)$ is a vector representing the change in the spacecraft’s state (position and velocity). Φ is the state transition matrix, which is used to a state at one time to a state at a later time. A is the equations of motion matrix, and contains equations relating the value of the state vector to the value of the change in state vector. A is also used to propagate the state transition matrix forward in time. P is the covariance matrix, and contains estimates of the inaccuracy in the filter’s prediction of each state variable.

The observation variables include the following. \mathbf{Y} is a vector containing the observed measurement. G is the observation function, which takes as input the reference trajectory and the current time value, and outputs the value of the expected measurement. \mathbf{y} is a vector storing the difference between \mathbf{Y} and G (the difference between the expected measurement and the observed measurement). \mathbf{y} is called the residual. \tilde{H} is the observation-state matrix, which is the partial derivative of G with respect to the state vector. R is the noise matrix, and is a user-defined constant. It consists of the noise levels that are predicted in the observed measurement. Finally, K is the gain matrix, which defines how much the filter weights the residual in its predictions.

Several notations also appear throughout the filter's steps. An asterisk (*) over a term means that it is a nominal or expected value. A caret (^) over a term means that it is an *a priori* (immediately preceding the measurement) estimate of the value. A bar (–) over a term means that it is an *a posteriori* (immediately succeeding the measurement) estimate of the value. A dot (·) over a term means that it is the derivative of the value. A tilde (~) over a term is merely an identifier, and doesn't indicate anything about the value.

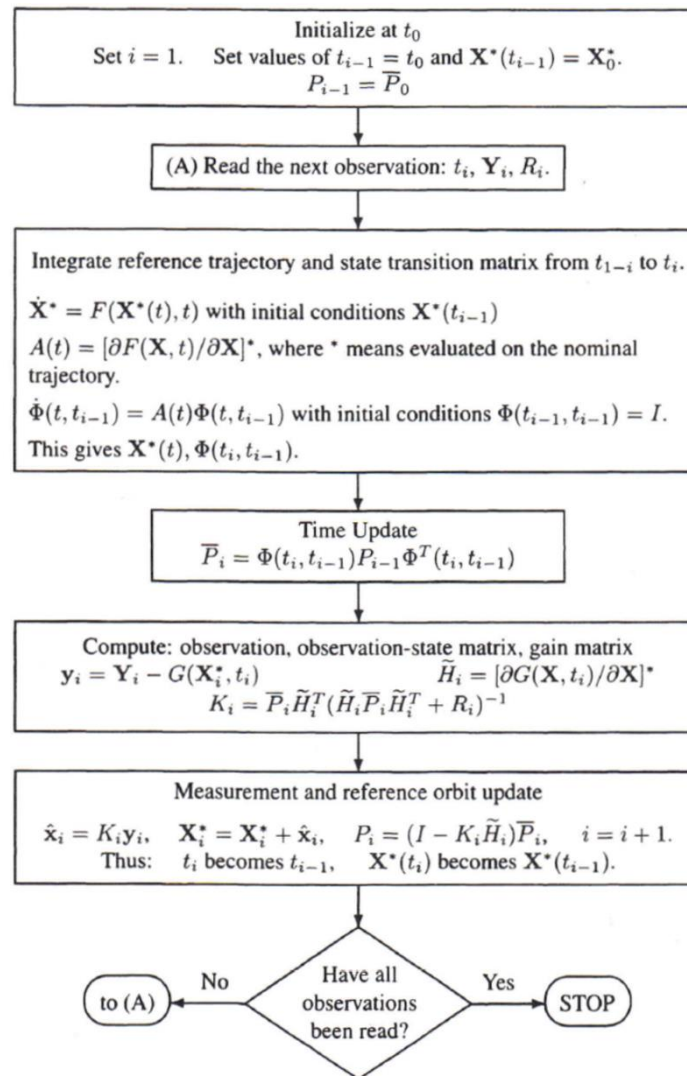


Figure 2.3: Extended Kalman Filter Flowchart⁹

As seen in the filter process, the state estimate \mathbf{X} is updated every time there is an observation. Assuming that the spacecraft is behaving approximately nominally, and isn't deviating significantly from predictions, every observation \mathbf{Y} causes the covariance P to decrease. Intuitively, this makes sense. If an observation has been taken indicating the position and velocity of the spacecraft, the uncertainty is low because there is recent evidence of the spacecraft's state. As time passes between observations, the uncertainty in knowledge about the spacecraft's state

builds up. Again, intuitively this makes sense. As time passes without an observation, random errors and noise have more time to build up and affect the state, and so there should be a larger uncertainty. This effect is why satellite operators would prefer to have constant measurements. With constant measurements, there is no time for random errors and noise, and therefore the uncertainty, to build up. However, this can be cost prohibitive.

On the other end of the spectrum, too few measurements can cause additional problems in the filter. If the measurements are very sparse, and the residuals become too large, the errors grow to a point where the equations of motion are unsolvable and the filter diverges. The point where the filter diverges is characterized by a sharp increase in the trace of the covariance matrix. Once the filter has diverged, new measurements do not affect the solution, and the problem becomes unsolvable until the filter is restarted.

A Kalman filter, in the end, provides an estimate of the true state of the spacecraft and a value of the uncertainty in that estimate. This uncertainty is used in the tracking efficiency calculation. By using the average value of the trace of the covariance matrix over the length of the mission, there is a direct way to compare the performance of each test case.

Chapter 3

Analysis

For this analysis, the process for identifying cost-effective tracking strategies under a range of constraints is demonstrated for a representative lunar landing mission. First, an overview of the process to be demonstrated is presented. Second, the Earth-Moon trajectory constructed for this demonstration in Systems Tool Kit (STK)^{††} is shown. Third, ODTK and its ability to accurately simulate and filter ground measurements is discussed. Fourth, the tracking schedule parameters used to represent a wide range of schedule types is presented, along with examples that demonstrate the flexibility of this simple representation. Fifth, the antenna configurations used in this demonstration are shown, along with notes about additional considerations that must be made by mission designers when selecting their own configurations to test. Sixth, the use of MATLAB to automate the analysis process is discussed. Last, the tracking efficiency, a measure of the cost-efficiency of a tracking solution, is shown, along with equations and examples.

Process Overview

The process that mission designers should follow to generate the desired end product consists of five main steps. First, the trajectory should be defined. In most cases, this will already have been done well before the analysis even begins, as that would be one of the first items generated in the design phase.

Second, the constraints in the selection of the tracking solution should be identified. For example, if the Deep Space Network only has time available on a limited number of antennas, then there would be constraints on the antenna configuration selection. Constraints should be

^{††} <http://www.agi.com/products/stk/>; accessed 8/10/14

categorized into hard constraints, where an exact value must be met, and soft constraints, where a range of values is acceptable. They should also be categorized by whether they affect an independent variable or a dependent variable, as that informs when the constraint is applied.

Third, the independent variables should be identified, and the range of potential values of those variables should be defined by the mission designer. Several discrete values from across the range should be selected as test cases. In order to identify patterns in the data, it is recommended that the values selected be fairly evenly spaced across the range.

Fourth, tracking solutions consisting of all combinations of these independent variable values must be tested in a simulator. The simulator must include a mock tracking data generator and an Extended Kalman Filter to process the tracking data. The goal is to calculate a realistic value of the performance of each tracking solution, as it would be used in operations.

Finally, the results of the simulation will be used to generate a measure of the cost-efficiency of the solution. This measure allows for comparison of the performance of all of the potential solutions.

A flow chart of the process to test an individual tracking solution is shown in Figure 3.1. For a case where there are many potential solutions, the process shown should be repeated for every solution allowed by the constraints.

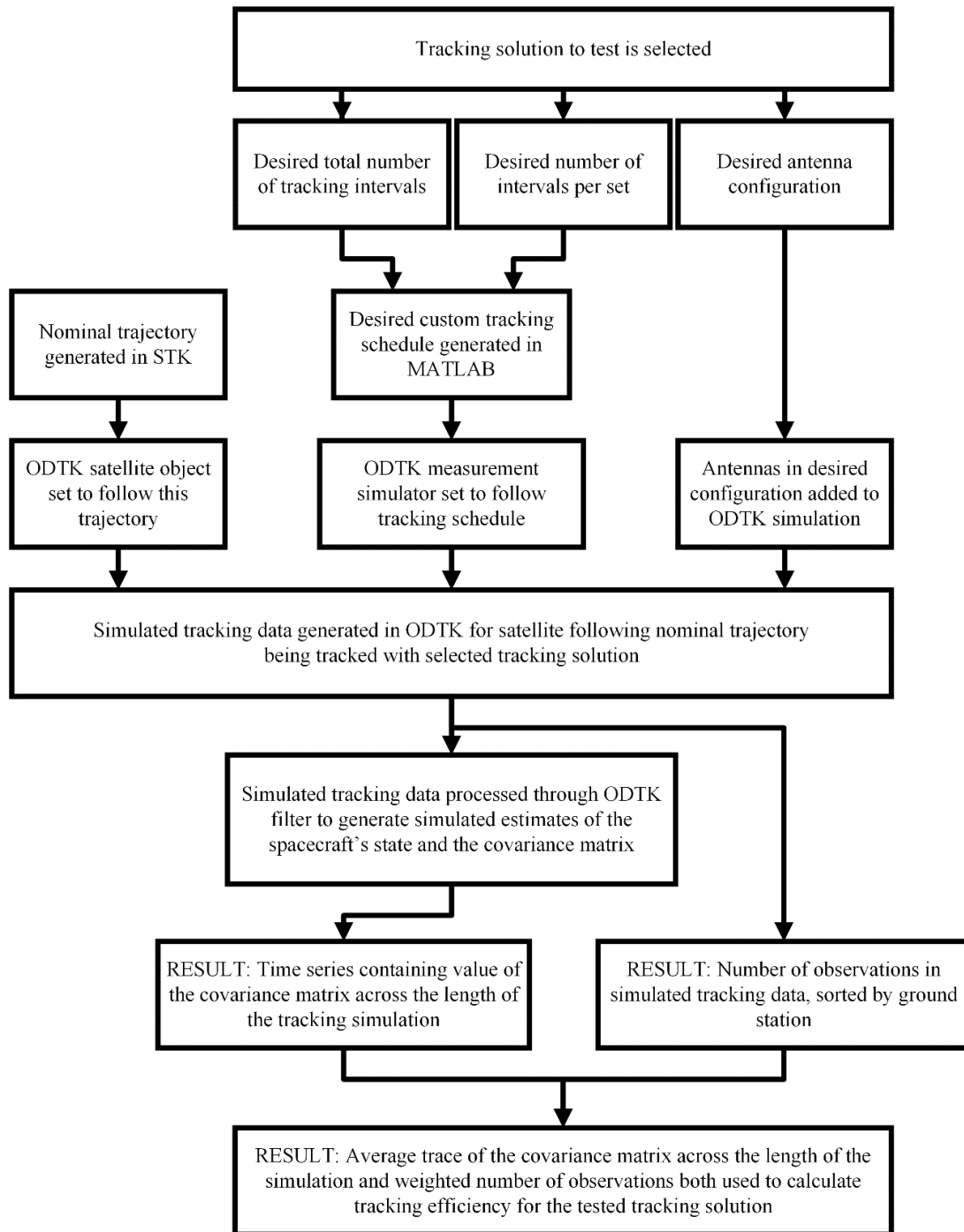


Figure 3.1: Flowchart of Overall Process

Nominal Trajectory Generation in Systems Tool Kit

In order to generate simulated measurements of a spacecraft, an ephemeris is required that provides the true state of the spacecraft over time. STK, along with the Astrogator add-on^{††}, is used to generate the orbit and then the ephemeris is exported to ODTK.

The example orbit generated for the analysis is an Earth-Moon transfer. The launch is from Kennedy Space Center in Florida on July 31, 2016. Immediately after launch, the spacecraft enters a low-inclination (~28 degrees), 300 kilometer altitude circular parking orbit. After 2.5 orbits, it executes a trans-lunar injection (TLI) maneuver near the ascending node, which puts the craft into a three day cruise phase to the Moon. This mission profile is similar to that of the Apollo missions¹⁰. However, unlike the Apollo missions which first entered a lunar parking orbit, this simulated spacecraft is placed on a direct descent trajectory, which means that the spacecraft does not go into orbit around the Moon prior to landing. The trajectory is designed using a similar methodology to Carrico and Loucks in Ref. 11. The significant orbital parameters are shown in Table 3.1.

All of the orbital parameters, including launch date, are selected arbitrarily and do not correspond to any particular mission. The goal is to generate a simple lunar transfer trajectory that did not incorporate any particularly unique features, so the analysis methods are applicable to a wide range of missions.

^{††} <http://www.agi.com/products/stk/modules/default.aspx/id/stk-astrogator>; accessed 11/24/14

Table 3.1: Mission Orbital Parameters

| <u>Mission Phase</u> | <u>Parameter</u> | <u>Value</u> |
|----------------------|---------------------------------------|---|
| Launch | Epoch | 31 Jul 2016 13:46:12 UTCG |
| Pre-TLI | Epoch | 31 Jul 2016 17:52:47 UTCG |
| | Semi-major axis | 6677.383 km |
| | Eccentricity | 0.00181 |
| | Inclination | 28.111 deg |
| | Right ascension of the ascending node | 346.804 deg |
| | Argument of periapsis | 322.285 deg |
| | True anomaly | 18.202 deg |
| TLI | Delta-V | 3.1396 km/s (impulsive) |
| Post-TLI | Epoch | 31 Jul 2016 17:52:47 UTCG |
| | Semi-major axis | 321571.03 km |
| | Eccentricity | 0.9793 |
| | Inclination | 28.111 deg |
| | Right ascension of the ascending node | 346.808 deg |
| | Argument of periapsis | 340.42 deg |
| | True anomaly | 0.0652 deg |
| Landing | Epoch | 3 Aug 2016 09:32:06 UTCG (stopped on lunar impact) |

Measurement Simulation in Orbit Determination Tool Kit

Orbit Determination Tool Kit (ODTK) is the software used in this thesis to simulate each potential tracking solution. Though STK is very capable at designing trajectories, its ability to accurately simulate measurements is not as developed as those in Orbit Determination Tool Kit (ODTK).

A useful feature of the program for this analysis is the ODTK Facility Database. This digital repository of hundreds of different ground stations from around the world, including those of the Deep Space Network, includes information about the properties and restrictions of each station. Allowable measurement types, noise levels, drift rates, minimum and maximum slew rates and elevations for tracking are all examples of properties included in a ground station's file. ODTK uses these values when it is simulating measurements from a ground station in order to

generate accurate simulations of tracking data from that station. For example, when simulated measurements are generated using a ground station object in ODTK, measurements that are outside the capabilities of the antennas (for example, the spacecraft is below the station's minimum elevation or traveling faster across the sky than the antenna can slew) are not created.

After the simulated tracking data is generated, it is processed through an ODTK filter. The ODTK filter is based on an Extended Kalman Filter¹². Several parameters of the filter are user defined, including the initial values for the uncertainty in each element of the orbit's state. As these initial values affect the final results of the thesis, their values are presented here. The values used are the default values for new "Satellite" objects in ODTK and are shown in Table 3.2. As with the orbital parameters, the uncertainties are selected arbitrarily and do not correspond to any particular mission.

Table 3.2: Initial Position and Velocity Uncertainties (1σ)

| Variable | Value |
|----------------------------------|--------------|
| Radial Position Uncertainty | 50 m |
| In-Track Position Uncertainty | 100 m |
| Cross-Track Position Uncertainty | 20 m |
| Radial Velocity Uncertainty | 0.06 m/s |
| In-Track Velocity Uncertainty | 0.04 m/s |
| Cross-Track Velocity Uncertainty | 0.02 m/s |

Generation of Custom Tracking Schedules

To generate a custom tracking schedule, two parameters are used in this thesis to define the properties of the schedule: the total number of tracking intervals and number of intervals per interval set. These two variables are defined below. Examples of different tracking schedules are also presented to show how the values of the variables affect the structure of the schedule. Recall

that for interplanetary spacecraft, tracking data is extracted from the spacecraft's communications signal, and dedicated tracking passes do not exist. Though this thesis uses the terminology "tracking schedule", in reality they are communications schedules.

Tracking Schedule Parameters

To facilitate the discussion of the analysis, several terms are now defined. The term "tracking interval" is defined as a period of time the length of a single observation during which tracking is turned on. For any individual station taking observations of a satellite, it takes exactly one observation per tracking interval. Note that this does not mean that there is only one observation per tracking interval. The number of observations taken per tracking interval is a function of the number of antennas that have a view of the satellite at that time. If a satellite is in the view of three antennas during one tracking interval, and all three antennas are being used for tracking, then three observations are taken (one per antenna).

The term "interval set" is defined as a group of tracking intervals. As an example, a two-interval tracking set is an interval that is two measurements in length. Each custom tracking schedule consists of a number of tracking sets that are equally spaced in time across the entire mission.

The term "gap in coverage" is defined as a period of time between any two consecutive interval sets. When a custom tracking schedule is being generated, MATLAB calculates the length of the gaps in coverage in such a way that all of the gaps are the same length. This calculation is shown in the next section.

Tracking Schedule Calculations

Using the selected tracking schedule parameters, several other variables defining the schedule are calculated using Equations (3.1) through (3.4). In all of the equations, numIntervals is the total number of tracking intervals, numSets is the total number of interval sets, numGaps is the total number of gaps in coverage, timePerInterval is the time per tracking interval, timePerGap is the time per gap in coverage, timePerSet is the time per interval set, numIntervalsPerSet is the number of intervals per interval set, and totalMissionLength is the total length of the mission (for the sample scenario to be analyzed in this thesis, totalMissionLength is equal to 229159 seconds)

$$\text{numSets} = \frac{\text{numIntervals}}{\text{numIntervalsPerSet}} \quad (3.1)$$

$$\text{numGaps} = \text{numSets} - 1 \quad (3.2)$$

$$\text{timePerSet} = \text{timePerInterval} \times \text{numIntervalsPerSet} \quad (3.3)$$

$$\text{timePerGap} = \frac{\text{totalMissionLength} - \text{numIntervals} \times \text{timePerInterval}}{\text{numGaps}} \quad (3.4)$$

Tracking Schedule Examples

Several examples of tracking schedules based on various parameters are shown graphically in Figure 3.2. In each schedule, a gray rectangle corresponds to a tracking interval, and the parameters defining the schedule are shown on the right hand side. In this figure, it is seen how the values of the tracking schedule parameters affect the structure of the generated tracking

schedule, and how the gaps in coverage are calculated in such a way that the interval sets are equally spaced across the mission length.

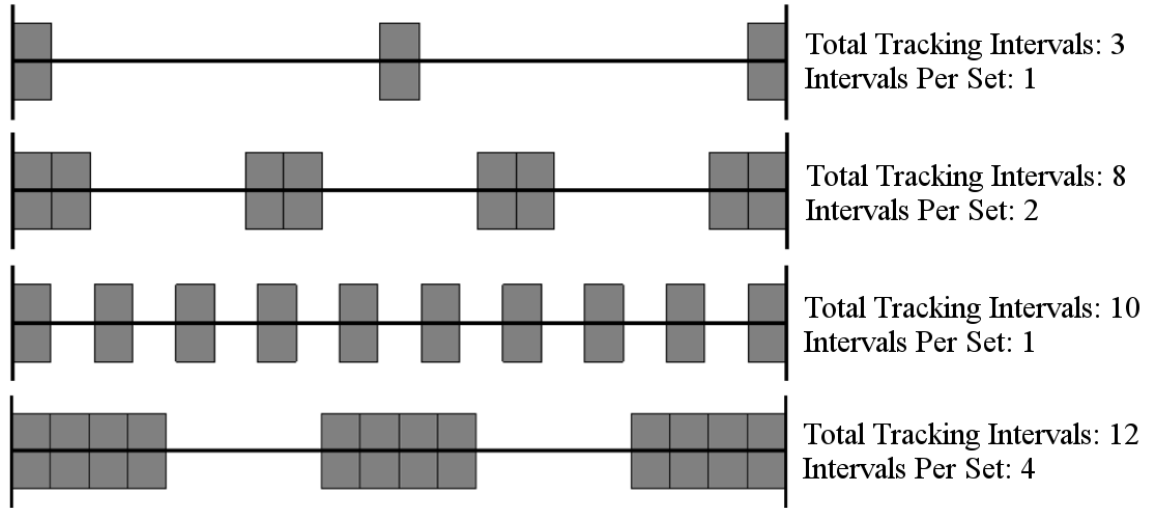


Figure 3.2: Examples of Custom Tracking Schedules

Several parameters are the same across all the tracking schedules. Each observation, and therefore each tracking interval, is arbitrarily defined as taking 10 seconds. Every schedule starts at time 0 s, which corresponds to immediately after TLI, and ends at time 229159 s, which corresponds to the lunar landing at the end of the mission. For any observation, there are no restrictions to which antennas can track the spacecraft for that individual observation. Allowable measurement types are sequential ranging, two-way Doppler and three-way Doppler.

Range of Values for Tracking Parameters

For the demonstration, ten different values of the total number of tracking intervals and eight different values for number of intervals per set are used. All of the test cases are shown in matrix form in Figure 3.3. Every dot in the chart indicates a separate tracking schedule. The rows

indicate the total number of tracking intervals combinations and the columns indicate the number of intervals per set.

| | | Number of Intervals per Set | | | | | | | |
|------------------------------------|-----|-----------------------------|---|---|---|---|---|----|----|
| | | 1 | 2 | 3 | 4 | 6 | 8 | 12 | 24 |
| Total Number of Tracking Intervals | 24 | ● | ● | ● | ● | ● | ● | ● | ● |
| | 48 | ● | ● | ● | ● | ● | ● | ● | ● |
| | 72 | ● | ● | ● | ● | ● | ● | ● | ● |
| | 96 | ● | ● | ● | ● | ● | ● | ● | ● |
| | 120 | ● | ● | ● | ● | ● | ● | ● | ● |
| | 144 | ● | ● | ● | ● | ● | ● | ● | ● |
| | 168 | ● | ● | ● | ● | ● | ● | ● | ● |
| | 192 | ● | ● | ● | ● | ● | ● | ● | ● |
| | 216 | ● | ● | ● | ● | ● | ● | ● | ● |
| | 240 | ● | ● | ● | ● | ● | ● | ● | ● |

Figure 3.3: Range of Tracking Schedules Tested

For this demonstration, multiples of 24 are chosen for the total number of tracking periods because they are evenly divisible by several denominators. This allows many different options for number of intervals per set, and therefore allows for numerous tracking schedule structures to be tested.

Antenna Configurations

Five different antenna configurations are tested in the analysis. All of them are “symmetric” configurations, which is defined in this thesis as configurations where the same number and type of antennas are being used at each of the DSN sites (e.g. one 34-meter dish and

one 70-meter dish per site). The five configurations that are tested are shown visually in Table 3.3. A dot in a cell means that the antenna of that column is turned on and used for tracking when testing the configuration of that row.

Table 3.3: Antenna Configurations for Analysis

| | | Goldstone 34m #1 (DSS-26) | Goldstone 34m #2 (DSS-24) | Goldstone 70m (DSS-14) | Canberra 34m #1 (DSS-34) | Canberra 34m #2 (DSS-45) | Canberra 70m (DSS-43) | Madrid 34m #1 (DSS-54) | Madrid 34m #2 (DSS-55) | Madrid 70m (DSS-63) | Configuration Description |
|-----------|-----------------|---------------------------|---------------------------|------------------------|--------------------------|--------------------------|-----------------------|------------------------|------------------------|---------------------|-----------------------------|
| Symmetric | Configuration 1 | ● | ● | ● | ● | ● | ● | ● | ● | ● | All sites: One 70m, two 34m |
| | Configuration 2 | ● | | ● | ● | | ● | ● | | ● | All sites: One 70m, one 34m |
| | Configuration 3 | | | ● | | | ● | | | ● | All sites: One 70m |
| | Configuration 4 | ● | ● | | ● | ● | | ● | ● | | All sites: Two 34m |
| | Configuration 5 | ● | | | ● | | | ● | | | All sites: One 34m |

When a mission designer is choosing antenna configurations to test when identifying cost-effective tracking solutions, they will be able to choose whichever configurations they are the most likely to use. For this analysis, several were chosen arbitrarily as example cases.

“Asymmetric” antenna configurations, where the number and type of antennas used are not the same at each location, are not used in this analysis. They can be tested if desired, though care must be taken to ensure that the results of any of the asymmetric cases are not overgeneralized. If an asymmetric configuration were used for the trajectory in this thesis, the effect of selecting any particular site as the “odd one out” would be minimal. However, this is not always the case. For a highly inclined orbit, for example, the satellite may never be in view of some of the stations (e.g. a lunar transfer orbit departing over the North Pole means that the satellite does not have contact with Canberra, in the Southern Hemisphere, during the first part of its transfer). The mission designer should use their knowledge of the orbit to ensure that results of

one tested antenna configuration are not considered to be relevant for antenna configurations that appear to have a similar layout.

Process Automation with MATLAB

The majority of the analysis is automated with MATLAB. For every test case, after the STK ephemeris is built, MATLAB begins by generating and setting up an ODTK scenario. First, all of the DSN antennas in the test case's antenna configuration are imported into the scenario as tracking objects from the ODTK Facility Database. Next, a satellite object is created and added to the scenario. The satellite object's ephemeris is set to the ephemeris generated earlier in STK and the orbit uncertainty is set to the values in Table 3.2. A simulator object is then created and added to the scenario for each satellite. A custom tracking schedule is then generated in MATLAB that has the parameters of that test case (defined values for total number of tracking intervals and the number of intervals per set), and the schedule is paired with the simulator object. This ensures that simulated measurements follow that test case's custom tracking schedule. Finally, a filter object is created and added to the scenario. It processes the simulated measurements to calculate estimates of the spacecraft's state over the length of the mission.

After the simulator and filter are run, the results are exported from ODTK and processed in MATLAB. The number of observations taken by each facility are stored in a data file and processed to calculate a weighted observation count for that tracking solution (further described below). The results of the filter are also stored in a data file and then processed. The average values for each state variable's uncertainty over the length of the mission are calculated. Both the weighted measurement count and the average uncertainty are used in calculating the value of the tracking efficiency for each tracking solution.

Tracking Efficiency

In order to directly compare test cases, a value is created and named the “tracking efficiency”. The tracking efficiency value combines a tracking solution’s average state uncertainty and number of measurements taken of the spacecraft into one value and is a measure of the cost-effectiveness of the tracking schedule. A higher tracking efficiency indicates a tracking solution that achieved good performance (a lower average uncertainty) with less observations. A lower tracking efficiency indicates a tracking solution that achieved poor performance (a higher average uncertainty) with more observations. The higher the tracking efficiency for a tracking solution, the more efficiently the observations are used in estimating the spacecraft state, and the more cost-efficient the tracking solution is.

The uncertainty in the orbital state is quantified using the trace of the covariance matrix (as seen in the Extended Kalman Filter section defined as P). The covariance matrix for the orbital state is defined shown in Equation (3.5), where $P_{j,k}$ is the covariance of j with respect to k , R , I and C are the radial, in-track, and cross-track positions, and \dot{R} , \dot{I} , and \dot{C} are the radial, in-track, and cross-track velocities

$$P = \begin{bmatrix} P_{R,R} & P_{R,I} & P_{R,C} & P_{R,\dot{R}} & P_{R,\dot{I}} & P_{R,\dot{C}} \\ P_{I,R} & P_{I,I} & P_{I,C} & P_{I,\dot{R}} & P_{I,\dot{I}} & P_{I,\dot{C}} \\ P_{C,R} & P_{C,I} & P_{C,C} & P_{C,\dot{R}} & P_{C,\dot{I}} & P_{C,\dot{C}} \\ P_{\dot{R},R} & P_{\dot{R},I} & P_{\dot{R},C} & P_{\dot{R},\dot{R}} & P_{\dot{R},\dot{I}} & P_{\dot{R},\dot{C}} \\ P_{\dot{I},R} & P_{\dot{I},I} & P_{\dot{I},C} & P_{\dot{I},\dot{R}} & P_{\dot{I},\dot{I}} & P_{\dot{I},\dot{C}} \\ P_{\dot{C},R} & P_{\dot{C},I} & P_{\dot{C},C} & P_{\dot{C},\dot{R}} & P_{\dot{C},\dot{I}} & P_{\dot{C},\dot{C}} \end{bmatrix} \quad (3.5)$$

The trace of the covariance matrix is calculated with Equation (3.6)

$$\text{tr}(P) = P_{R,R} + P_{I,I} + P_{C,C} + P_{\dot{R},\dot{R}} + P_{\dot{I},\dot{I}} + P_{\dot{C},\dot{C}} \quad (3.6)$$

As opposed to using the uncertainty in any one component of the state vector to determine the performance of a tracking solution, the trace of the covariance matrix provides a more holistic measure of the uncertainty in the Kalman filter's estimation of the spacecraft state.

The second part of the tracking efficiency calculation, the total number of observations taken of the spacecraft, is a weighted calculation based on antenna size using the weighting scheme discussed in the Deep Space Network section of Chapter 2. Observations taken with 70-meter dishes contribute four times as much as observations taken with 34-meter dishes. This calculation is shown in Equation (3.7), where $numObs_{Weighted}$ is the weighted observation count, $numObs_{34-meter}$ is the number of observations taken by 34-meter dishes, and $numObs_{70-meter}$ is the number of observations taken by 70-meter dishes

$$numObs_{Weighted} = numObs_{34-meter} + 4 * numObs_{70-meter} \quad (3.7)$$

Using both of the values calculated above, the tracking efficiency is calculated using Equation (3.8)

$$Tracking\ Efficiency = \frac{1}{numObs_{Weighted} * tr(P)_{ave}} \quad (3.8)$$

The tracking efficiency is defined as a function of a product of the two values so that neither value is “overpowering” in the calculation, as could happen if it was defined as a sum.

Chapter 4

Results

For the analysis, processes for identifying cost-efficient tracking solutions for a number of cases with particular constraints are shown for a representative Earth-Moon trajectory. As only one orbit is analyzed in this thesis, the specific results presented are not generalizable. However, the processes to identify cost-efficient solutions are applicable to any mission profile.

The process is shown for seven cases, each with different constraints. Case 1 covers when the total amount of tracking time is defined. Case 2 discusses when the number of intervals per set is defined. In Case 3, the antenna configuration is defined. Case 4 demonstrates the process when nothing is defined and there are no constraints. Case 5 covers when there is a defined maximum covariance. Case 6 shows how to select the best tracking solution when there is a defined maximum tracking cost. Finally, Case 7 considers a defined maximum length of the gaps in coverage.

Case 1: Defined Total Amount of Tracking Time

This case discusses the process for a scenario where the total amount of tracking time is defined. Therefore, this case is an optimization problem across two variables (number of intervals per set and antenna configuration) with one hard constraint (the total amount of tracking time). Each tracking interval has a duration of 10 seconds. In this analysis, the total amount of tracking time for each solution is set equal to 10 times the weighted number of observations.

Process to Identify Most Cost-Efficient Solution

Ten different examples of this case are presented, one for each amount of total tracking time considered in this thesis. The first example is for a total tracking time of 2400 seconds. Tracking simulations for every potential tracking solution using this tracking time are run. As there are five different antenna configurations and eight different values for intervals per set that can be used with the tracking time, forty different solutions must be tested. Then, the tracking efficiency of all the potential solutions are compared.

The results here must be shown three-dimensionally. For the easiest readability, heat maps are chosen to display the three-dimensional results. The results of this first example are presented in Figure 4.1.

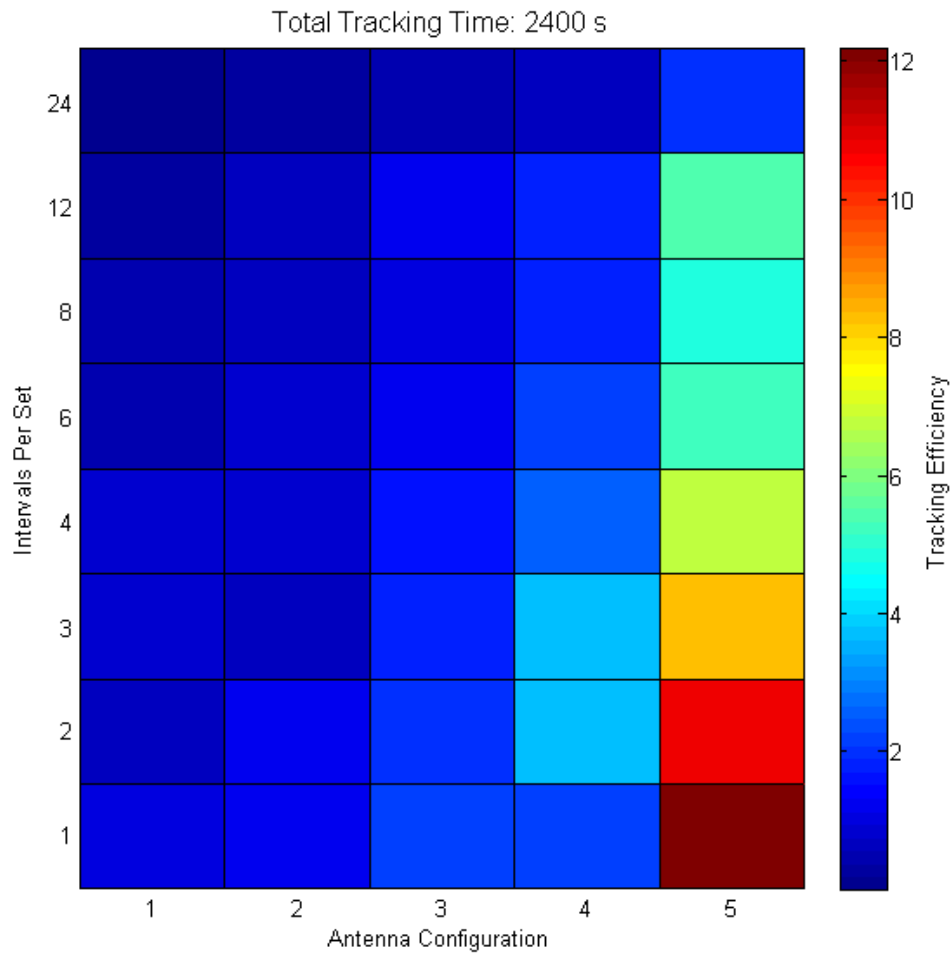


Figure 4.1: Tracking Efficiency vs. Antenna Configuration vs. Intervals per Set for Total Tracking Time of 2400 s

Each section of the heat map corresponds to a different potential solution. For this case, each column corresponds to an antenna configuration, and each row corresponds to a number of intervals per set. The color of each section corresponds to the tracking efficiency achieved. The most cost-efficient tracking solution can be identified by selecting the section with the highest tracking efficiency. For this particular trajectory and total tracking time, the solution using Antenna Configuration 5 and grouping the intervals in sets of 1 offers the highest tracking efficiency.

The second example is for a tracking schedule with 4800 seconds of total tracking time. The process for this example is the same as the process for the first. There are again forty possible solutions, and the section with the highest tracking efficiency is selected. The results for this example are presented in Figure 4.2. For this particular trajectory and total tracking time, the solution using Antenna Configuration 5 and grouping the intervals in sets of 1 offers the highest tracking efficiency.

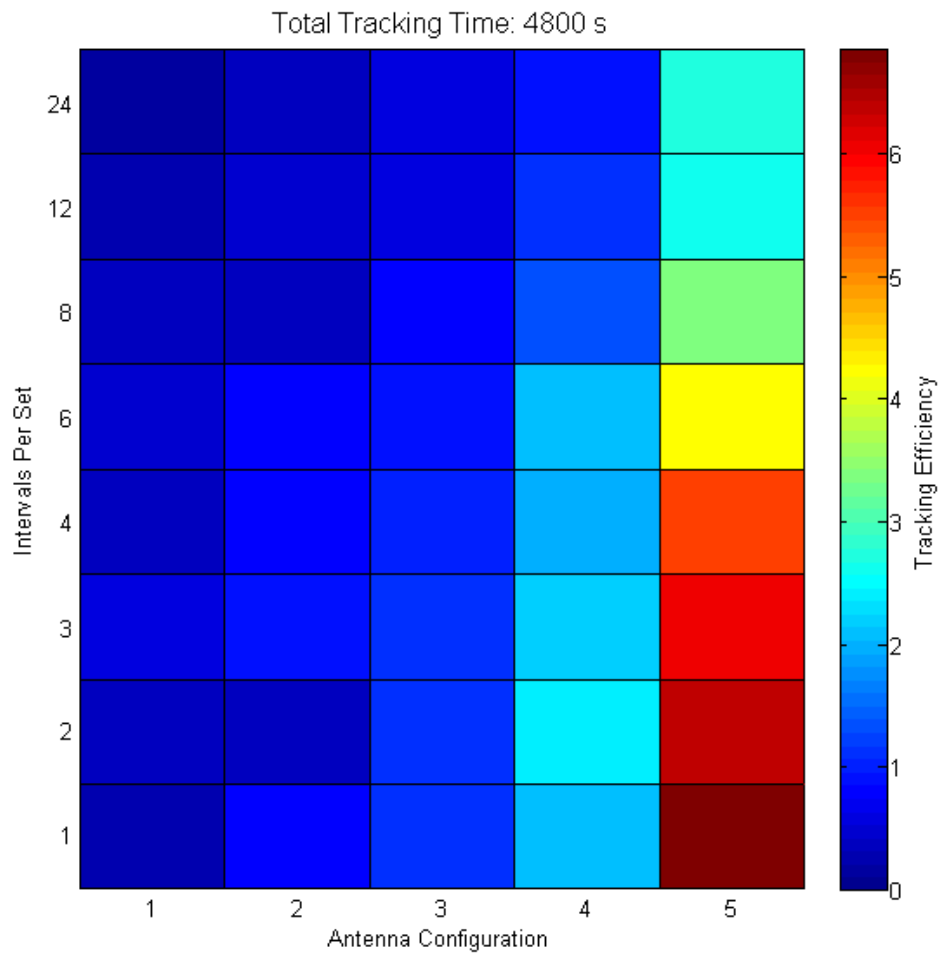


Figure 4.2: Tracking Efficiency vs. Antenna Configuration vs. Intervals per Set for Total Tracking Time of 4800 s

The third example is for a tracking schedule with 7200 seconds of total tracking time. The results for this example are presented in Figure 4.3. For this particular trajectory and total tracking time, the solution using Antenna Configuration 5 and grouping the intervals in sets of 1 offers the highest tracking efficiency.

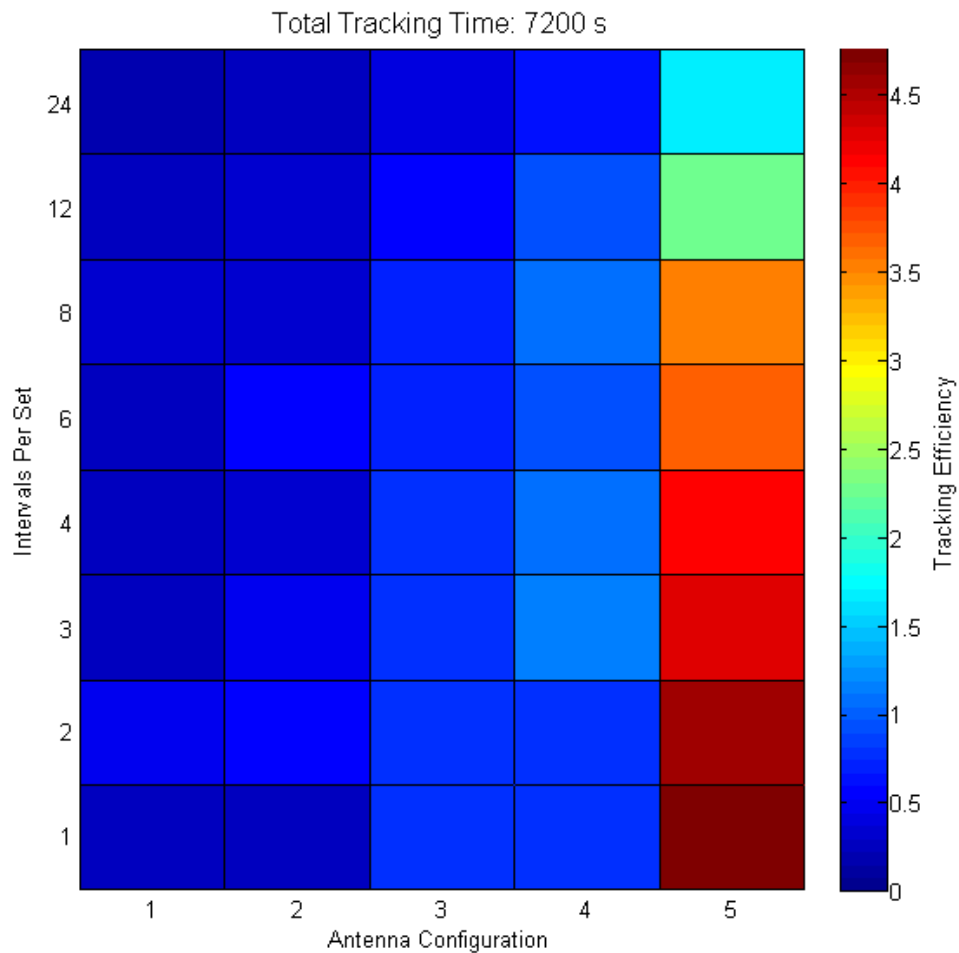


Figure 4.3: Tracking Efficiency vs. Antenna Configuration vs. Intervals per Set for Total Tracking Time of 7200 s

The fourth example is for a tracking schedule with 9600 seconds of total tracking time. The results for this example are presented in Figure 4.4. For this particular trajectory and total tracking time, the solution using Antenna Configuration 5 and grouping the intervals in sets of 1 offers the highest tracking efficiency.

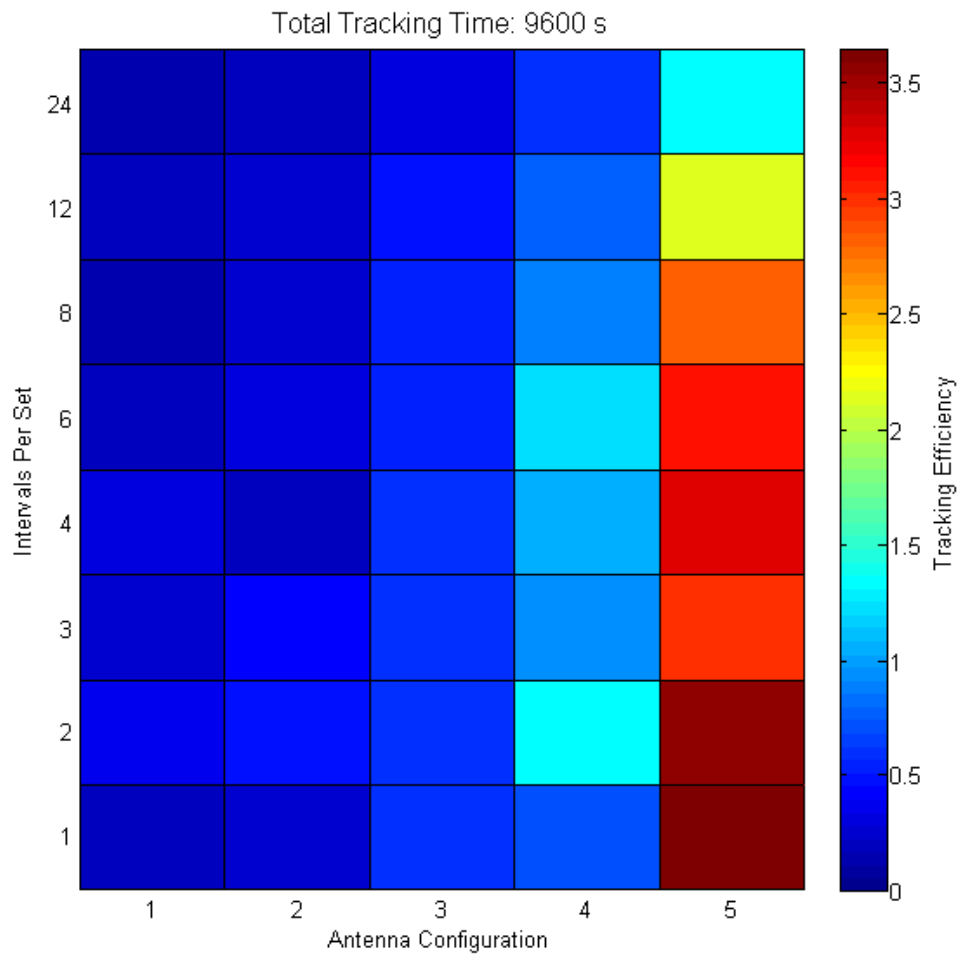


Figure 4.4: Tracking Efficiency vs. Antenna Configuration vs. Intervals per Set for Total Tracking Time of 9600 s

The fifth example is for a tracking schedule with 12000 seconds of total tracking time. The results for this example are presented in Figure 4.5. For this particular trajectory and total tracking time, the solution using Antenna Configuration 5 and grouping the intervals in sets of 2 offers the highest tracking efficiency.

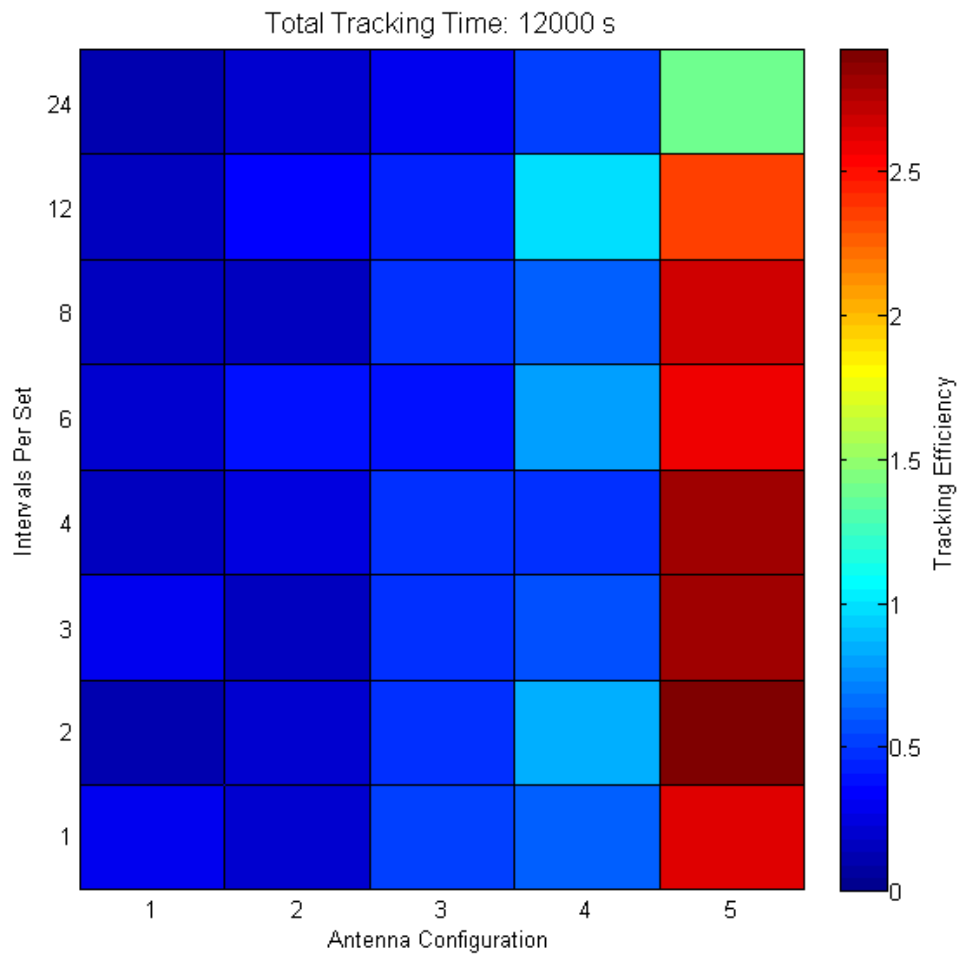


Figure 4.5: Tracking Efficiency vs. Antenna Configuration vs. Intervals per Set for Total Tracking Time of 12000 s

The sixth example is for a tracking schedule with 14400 seconds of total tracking time. The results for this example are presented in Figure 4.6. For this particular trajectory and total tracking time, the solution using Antenna Configuration 5 and grouping the intervals in sets of 1 offers the highest tracking efficiency.

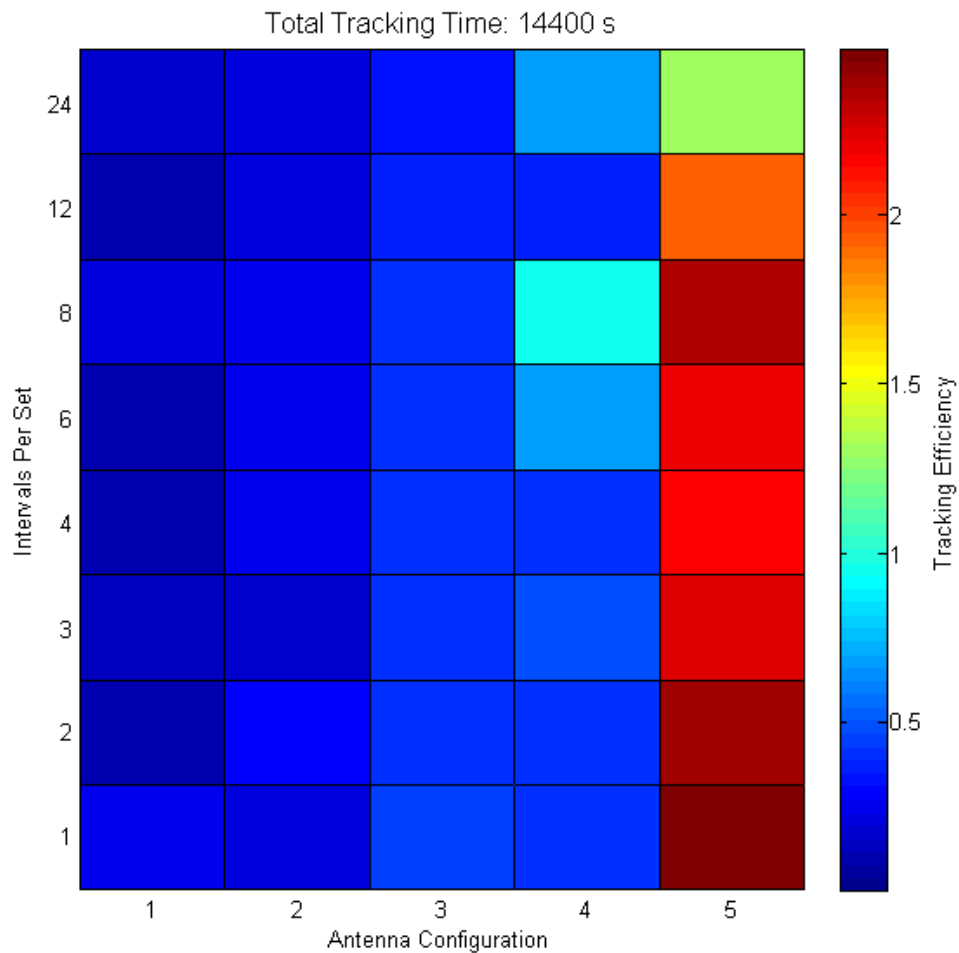


Figure 4.6: Tracking Efficiency vs. Antenna Configuration vs. Intervals per Set for Total Tracking Time of 14400 s

The seventh example is for a tracking schedule with 16800 seconds of total tracking time. The results for this example are presented in Figure 4.7. For this particular trajectory and total tracking time, the solution using Antenna Configuration 5 and grouping the intervals in sets of 1 offers the highest tracking efficiency.

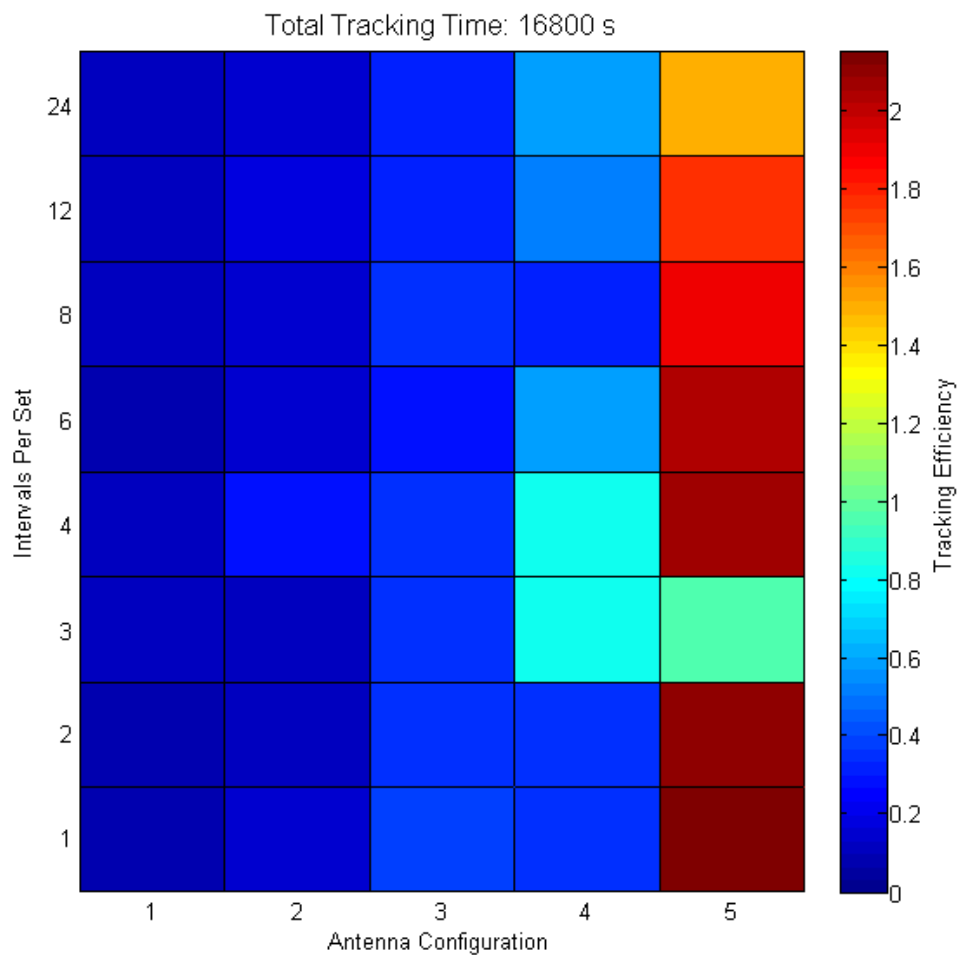


Figure 4.7: Tracking Efficiency vs. Antenna Configuration vs. Intervals per Set for Total Tracking Time of 16800 s

The eighth example is for a tracking schedule with 19200 seconds of total tracking time. The results for this example are presented in Figure 4.8. For this particular trajectory and total tracking time, the solution using Antenna Configuration 5 and grouping the intervals in sets of 4 offers the highest tracking efficiency.

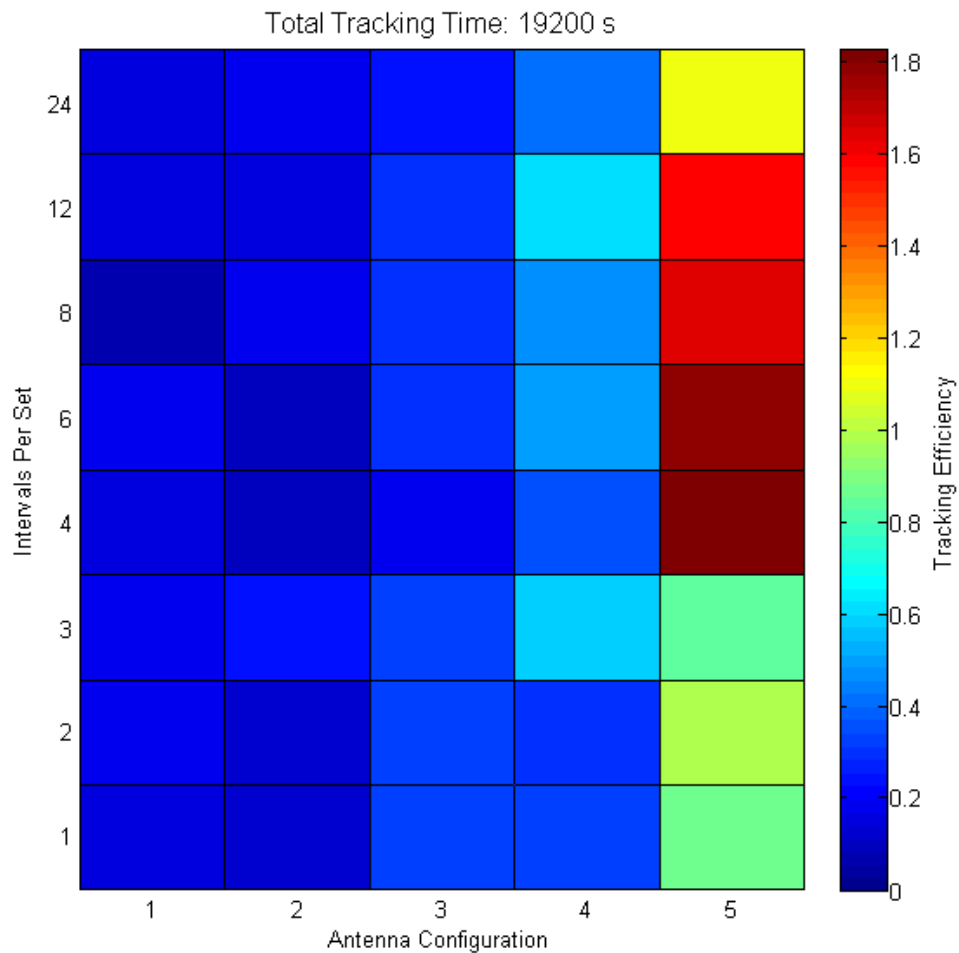


Figure 4.8: Tracking Efficiency vs. Antenna Configuration vs. Intervals per Set for Total Tracking Time of 19200 s

The ninth example is for a tracking schedule with 21600 seconds of total tracking time. The results for this example are presented in Figure 4.9. For this particular trajectory and total tracking time, the solution using Antenna Configuration 5 and grouping the intervals in sets of 1 offers the highest tracking efficiency.

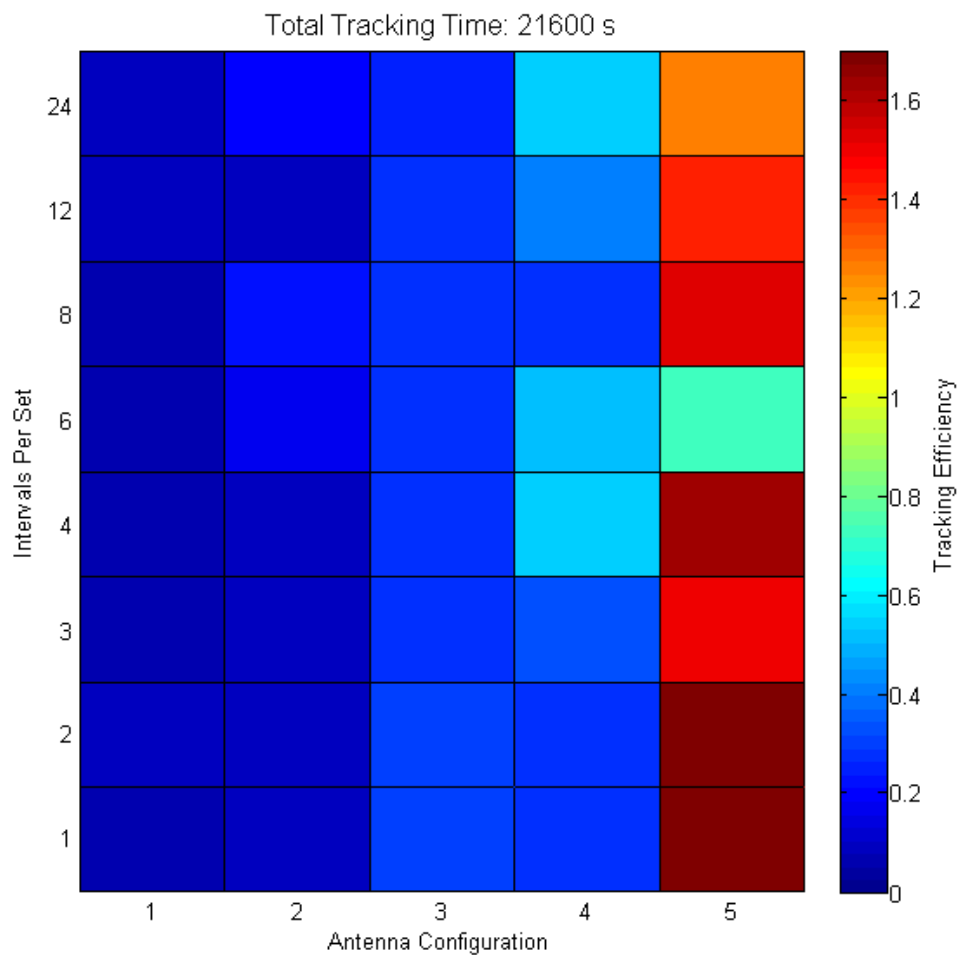


Figure 4.9: Tracking Efficiency vs. Antenna Configuration vs. Intervals per Set for Total Tracking Time of 21600 s

The tenth example is for a tracking schedule with 24000 seconds of total tracking time. The results for this example are presented in Figure 4.10. For this particular trajectory and total tracking time, the solution using Antenna Configuration 5 and grouping the intervals in sets of 1 offers the highest tracking efficiency.

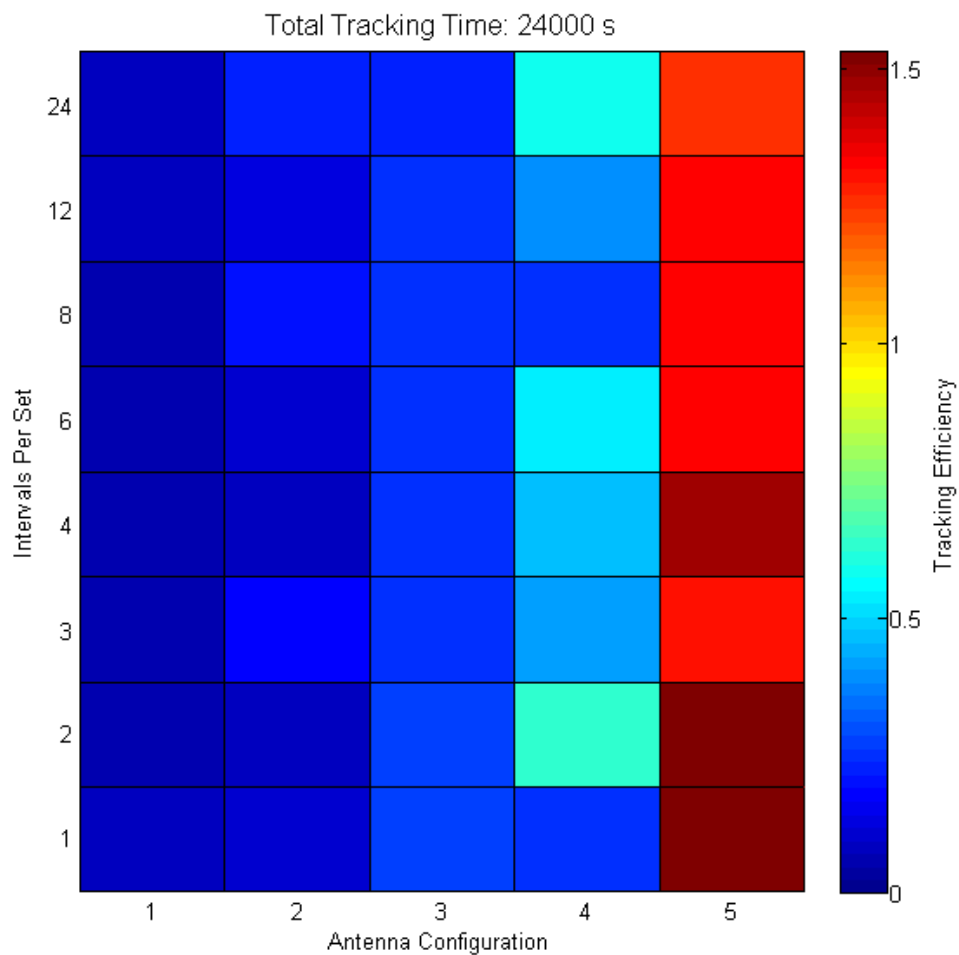


Figure 4.10: Tracking Efficiency vs. Antenna Configuration vs. Intervals per Set for Total Tracking Time of 24000 s

Sensitivity of Solution to Variables

The plots shown in the above examples provide much more information than just the most cost-efficient solution. The heat map format allows the mission designer to quickly view the sensitivity of the tracking efficiency to the independent variables. Each section on a plot represents a solution that is the combination of one constant and two variables. If the mission designer is restricted in his selection of a tracking solution and they are only allowed to select and define one of the two independent variables, they can look at the plot to determine which of the two variables is more important to cost-efficiency.

For this sample trajectory, and a total tracking time of 24000 seconds, the most cost-efficient solution uses Antenna Configuration 5 and intervals grouped in sets of 1. To determine the sensitivity of the cost-efficiency relative to each of the independent variables, the average performance of each column can be compared to the average performance of each row. The dimension with the higher average performance represents the variable to which the cost-efficiency is most sensitive. As can be seen in Figure 4.10, the average tracking efficiency across the solutions in the “Antenna Configuration 5” column is higher than the average across the solutions in the “1 Interval per Set” row. If a mission designer must decide between two solutions: one using Antenna Configuration 5 and not 1 interval per set, or one using 1 interval per set and not Antenna Configuration 5, they can look at the results and determine that, for a constant tracking time, the tracking efficiency is most strongly related to the antenna configuration. The designer should select the option using Antenna Configuration 5.

Robustness of Solution

Another analysis can be done using the results of the examples above to estimate the robustness of the selected solution. All of the figures above display the performance of all the possible solutions for each fixed amount of total tracking time. However, there may be cases where the exact amount of total tracking time is still defined, but it is subject to change. If a mission designer wants to select a tracking solution that is robust to changes in the defined amount of total tracking time, they can average the results of each of the individual total tracking time cases and identify the option with the best average performance.

For this sample trajectory, the results of this calculation are presented in Figure 4.11. The solution that is most robust to changes in the total tracking time is found by selecting the section with the highest average tracking efficiency across all the cases. In this case, the most robust solution uses Antenna Configuration 5 with intervals grouped in sets of 1.

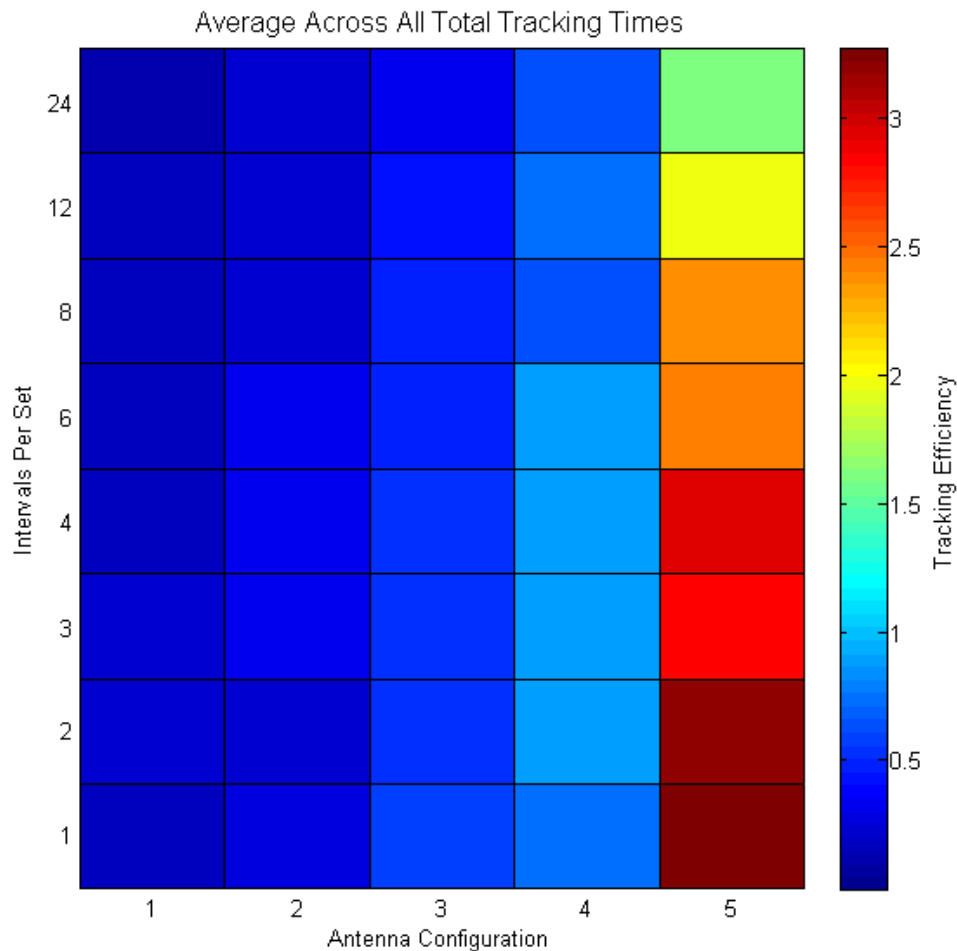


Figure 4.11: Tracking Efficiency vs. Antenna Configuration vs. Intervals per Set Averaged Across All Total Tracking Times

Summary

In summary, in the event that a total amount of tracking time is defined, mission designers can select the most cost-efficient tracking solution by testing all the possible solutions with that total tracking time. For this example, there are five antenna configurations and eight values for intervals per set, so there are forty solutions to test. However, a mission designer can select as many options for each of these variables as desired. The most cost-efficient tracking solution of the choices tested is the option with the highest tracking efficiency value.

In the event that the mission designer desires to use a different solution than the most cost-efficient option, they can look at the relative performance of all the solutions for a defined total tracking time to identify patterns that allow them to make cost-efficient decisions. For this example, it is seen from the results that the cost of using more than one 34-meter antenna per site is generally not worth the extra accuracy if selecting based on cost-efficiency. Additionally, for most of the tracking times tested, evenly spacing the tracking intervals across the length of the mission offers the most cost-efficient solution. A mission designer generating a tracking schedule for this mission would be able to see that, if they decided to deviate from the most cost-efficient solution, they will achieve the best performance on average by using Antenna Configuration 5 and/or intervals in sets of 1.

Case 2: Defined Number of Intervals per Set

This case discusses the process for a scenario where the number of intervals per set is defined. Therefore, this case is an optimization problem across two variables (total number of tracking intervals and antenna configuration) with one hard constraint (number of intervals per set).

Process to Identify Most Cost-Efficient Solution

Eight different examples of this case are presented, one for each of the possible number of intervals per set considered in this thesis. The first example is using 1 interval per set. Tracking simulations for every potential tracking solution using this number of intervals per set are run. As there are ten different values for total number of tracking intervals and five different values for antenna configuration that can be used with the number of intervals per set, fifty different

solutions must be tested. Then, the tracking efficiency of all the potential solutions are compared.

The results of this example are presented in Figure 4.12.

As in Case 1, the results must be shown three-dimensionally, and heat maps are again used. Each section of the heat map corresponds to a different potential solution. For this case, however, each row corresponds to a total number of tracking intervals, and each column corresponds to an antenna configuration. The color of each section corresponds to the tracking efficiency achieved. The most cost-efficient tracking solution can be identified by selecting the section with the highest tracking efficiency. For this particular trajectory and number of intervals per set, the solution using Antenna Configuration 5 and 240 total tracking periods offers the highest tracking efficiency.

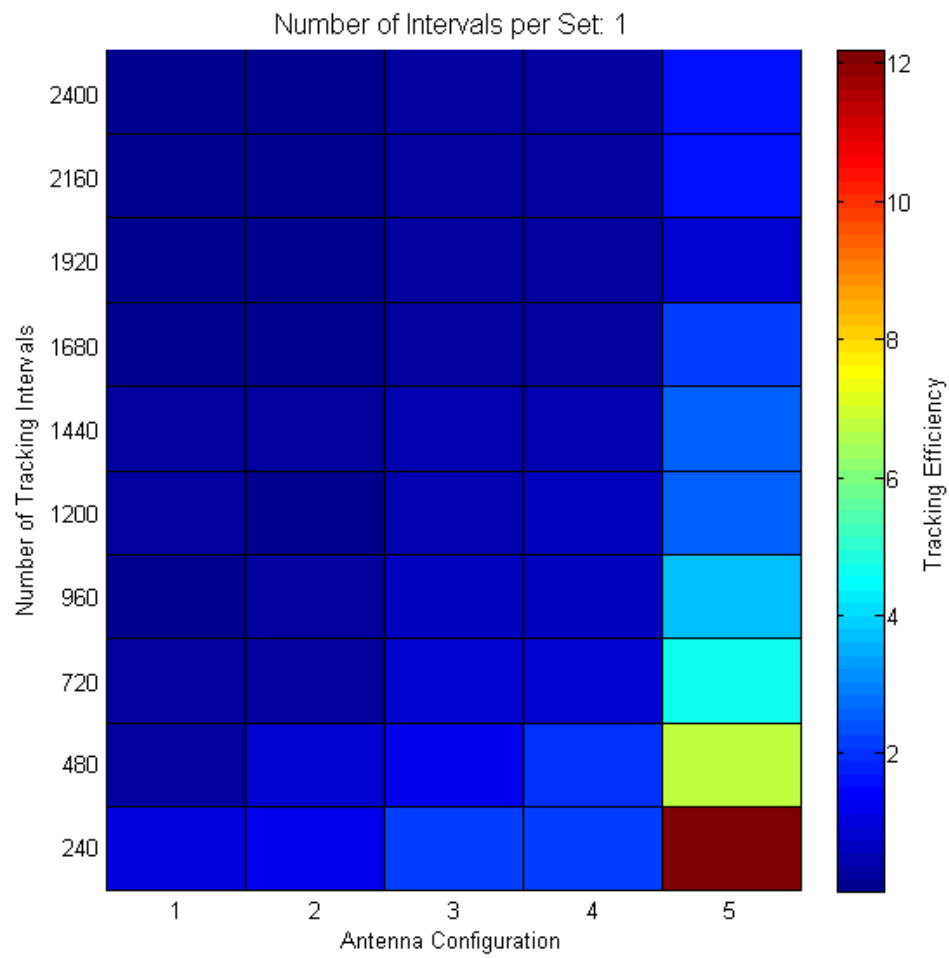


Figure 4.12: Tracking Efficiency vs. Antenna Configuration vs. Number of Tracking Intervals for Intervals in Sets of 1

The second example is using 2 intervals per set. The process for this example is the same as the process for the first. There are again fifty possible solutions, and the section with the highest tracking efficiency is selected. The results for this example are presented in Figure 4.13. For this particular trajectory and number of intervals per set, the solution using Antenna Configuration 5 and 240 total tracking periods offers the highest tracking efficiency.

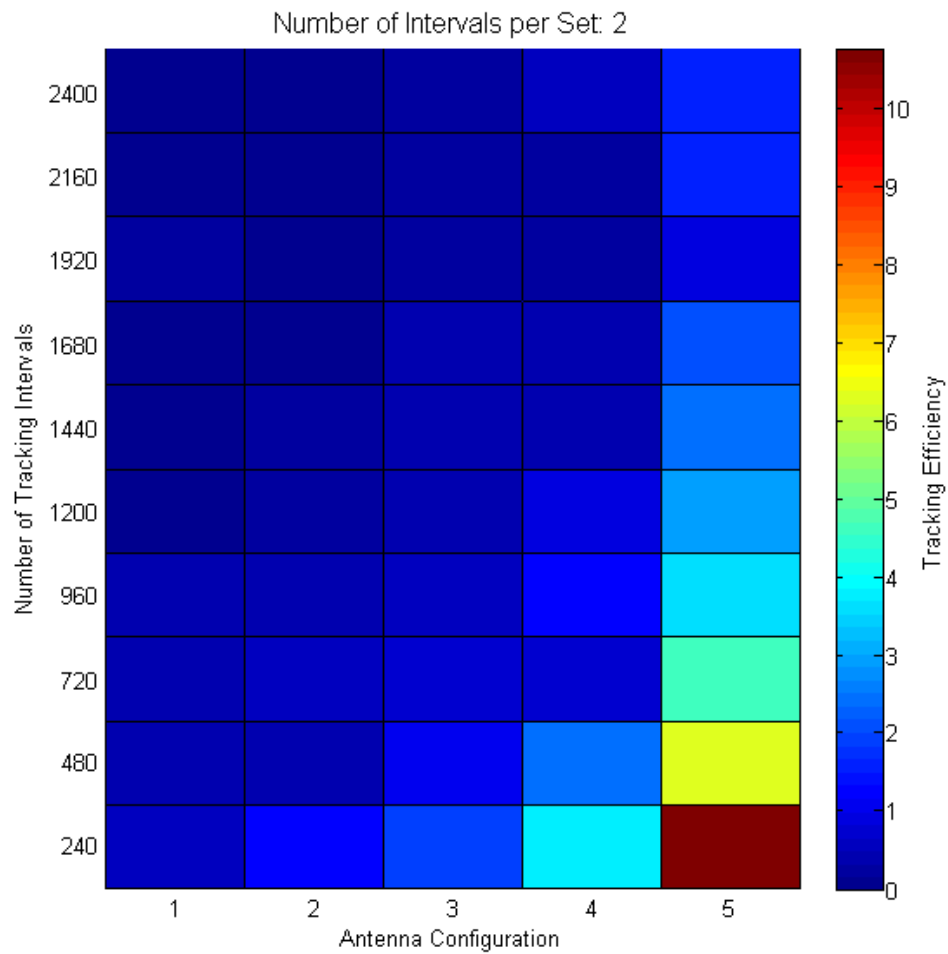


Figure 4.13: Tracking Efficiency vs. Antenna Configuration vs. Number of Tracking Intervals for Intervals in Sets of 2

The third example is using 3 intervals per set. The results for this example are presented in Figure 4.14. For this particular trajectory and number of intervals per set, the solution using Antenna Configuration 5 and 240 total tracking periods offers the highest tracking efficiency.

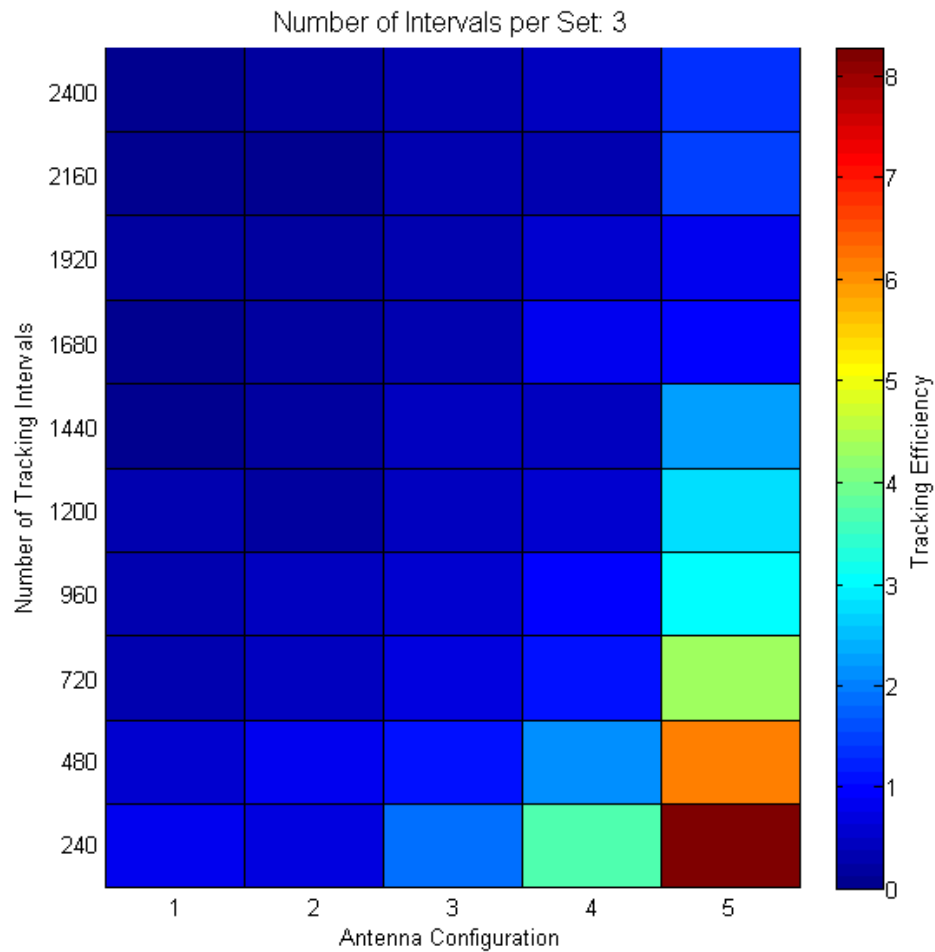


Figure 4.14: Tracking Efficiency vs. Antenna Configuration vs. Number of Tracking Intervals for Intervals in Sets of 3

The fourth example is using 4 intervals per set. The results for this example are presented in Figure 4.15. For this particular trajectory and number of intervals per set, the solution using Antenna Configuration 5 and 240 total tracking periods offers the highest tracking efficiency.

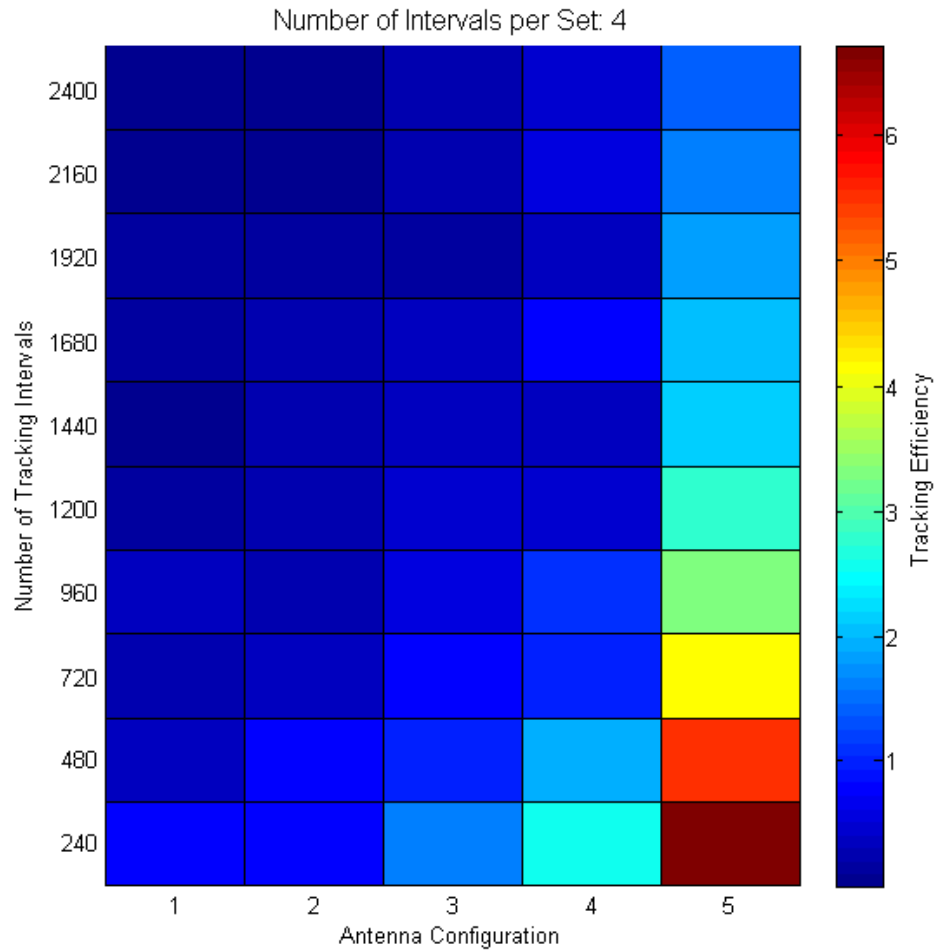


Figure 4.15: Tracking Efficiency vs. Antenna Configuration vs. Number of Tracking Intervals for Intervals in Sets of 4

The fifth example is using 6 intervals per set. The results for this example are presented in Figure 4.16. For this particular trajectory and number of intervals per set, the solution using Antenna Configuration 5 and 240 total tracking periods offers the highest tracking efficiency.

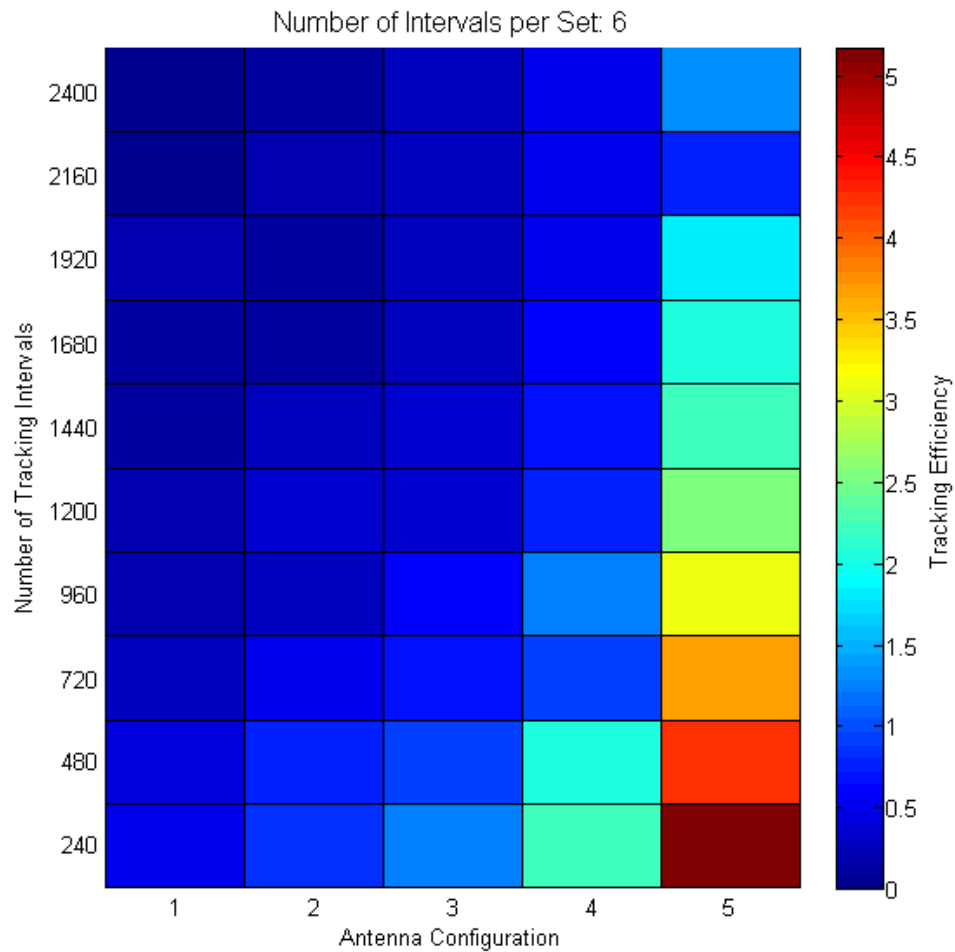


Figure 4.16: Tracking Efficiency vs. Antenna Configuration vs. Number of Tracking Intervals for Intervals in Sets of 6

The sixth example is using 8 intervals per set. The results for this example are presented in Figure 4.17. For this particular trajectory and number of intervals per set, the solution using Antenna Configuration 5 and 240 total tracking periods offers the highest tracking efficiency.

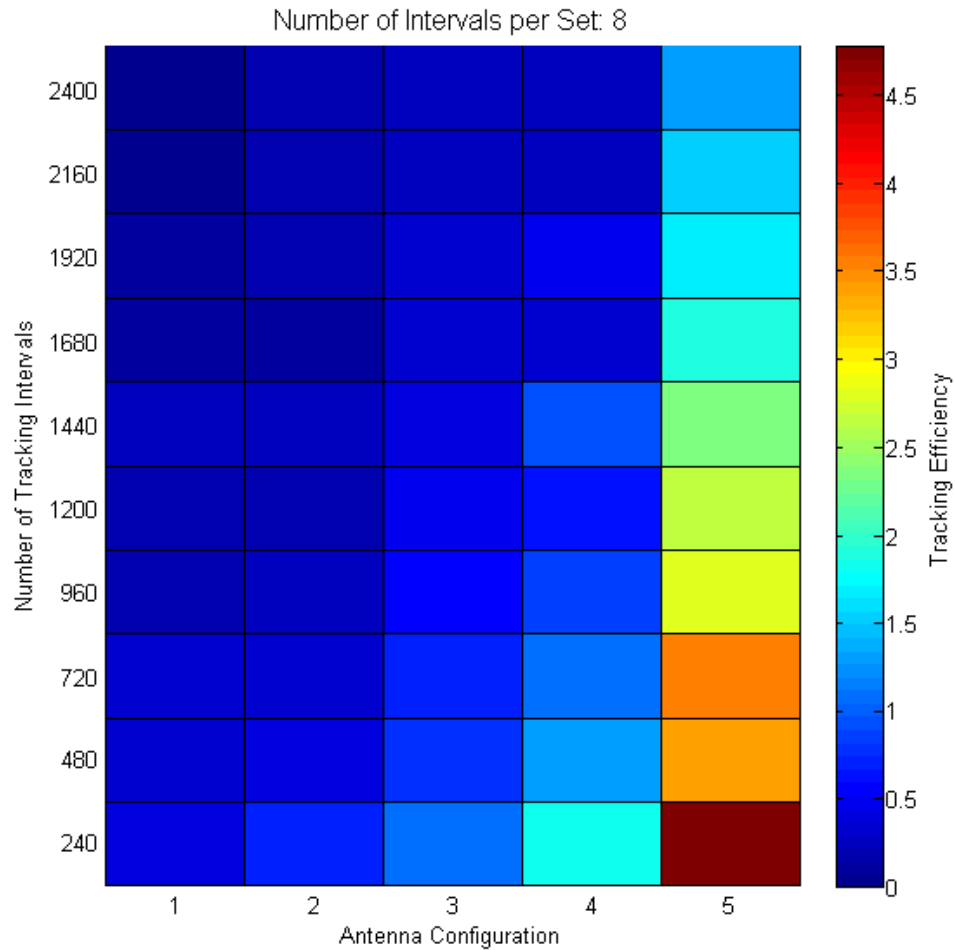


Figure 4.17: Tracking Efficiency vs. Antenna Configuration vs. Number of Tracking Intervals for Intervals in Sets of 8

The seventh example is using 12 intervals per set. The results for this example are presented in Figure 4.18. For this particular trajectory and number of intervals per set, the solution using Antenna Configuration 5 and 240 total tracking periods offers the highest tracking efficiency.

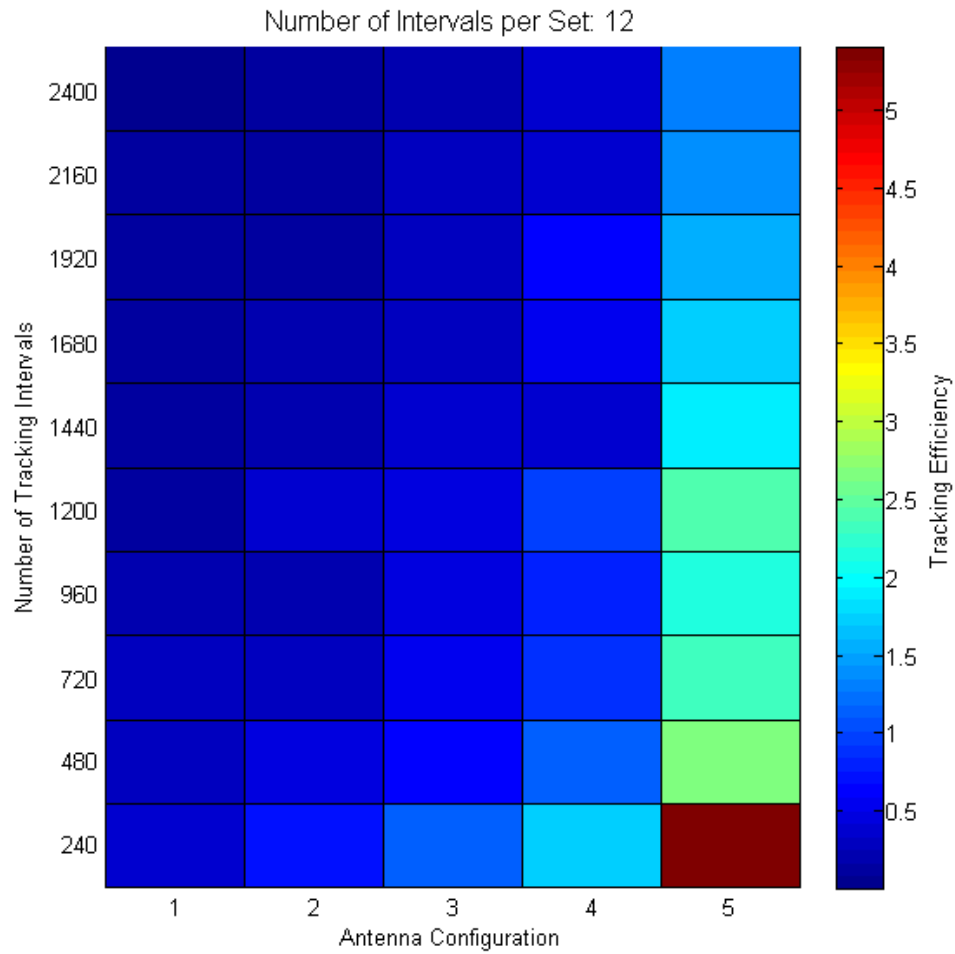


Figure 4.18: Tracking Efficiency vs. Antenna Configuration vs. Number of Tracking Intervals for Intervals in Sets of 12

The eighth example is using 24 intervals per set. The results for this example are presented in Figure 4.19. For this particular trajectory and number of intervals per set, the solution using Antenna Configuration 5 and 480 total tracking periods offers the highest tracking efficiency.

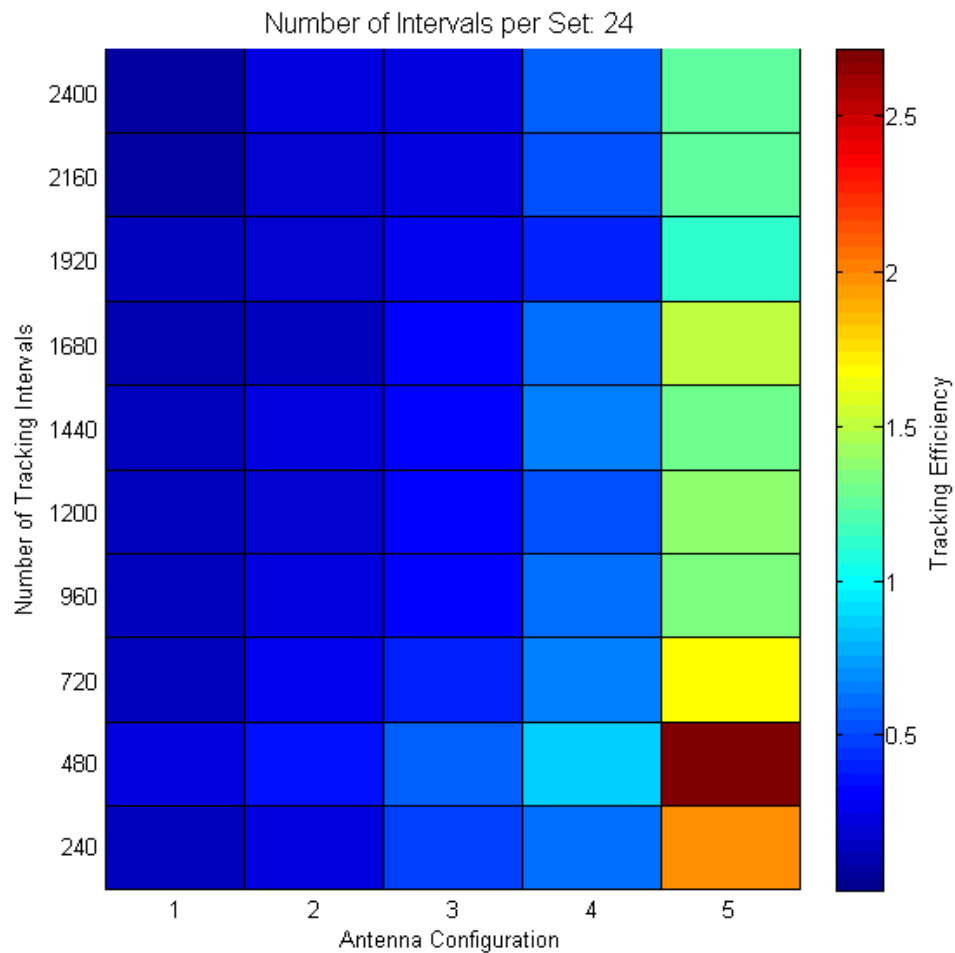


Figure 4.19: Tracking Efficiency vs. Antenna Configuration vs. Number of Tracking Intervals for Intervals in Sets of 24

Sensitivity of Solution to Variables

For this sample trajectory, and intervals grouped in sets of 8, the most cost-efficient solution uses Antenna Configuration 5 and 240 total tracking intervals. To determine the sensitivity of the cost-efficiency relative to each of the independent variables, the average performance of each column can be compared to the average performance of each row. The dimension with the higher average performance represents the variable to which the cost-efficiency is most sensitive. As can be seen in Figure 4.17, the average tracking efficiency across the solutions in the “Antenna Configuration 5” column is higher than the average across the solutions in the “240 Total Tracking Intervals” row. If a mission designer is forced to decide between two solutions: one using Antenna Configuration 5 and not 240 total tracking intervals, or one using 240 total tracking intervals and not Antenna Configuration 5, they can look at the results and determine that, for a constant number of intervals per set, the tracking efficiency is most strongly related to the antenna configuration. The designer should select the option using Antenna Configuration 5.

Robustness of Solution

Another analysis can be done using the results of the examples above to estimate the robustness of the selected solution. All of the figures above display the performance of all the possible solutions for each fixed value for number of intervals per set. However, there may be cases where the number of intervals per set is still defined, but it is subject to change. If a mission designer wants to select a tracking solution that is robust to changes in the defined number of intervals per set, they can average the results of each of the individual number of intervals per set cases and identify the option with the best average performance.

For this sample trajectory, the results of this calculation are presented in Figure 4.20. The solution that is most robust to changes in the number of intervals per set is found by selecting the section with the highest average tracking efficiency across all the cases. In this case, the most robust solution across all potential numbers of intervals per set uses 240 total intervals and Antenna Configuration 5.

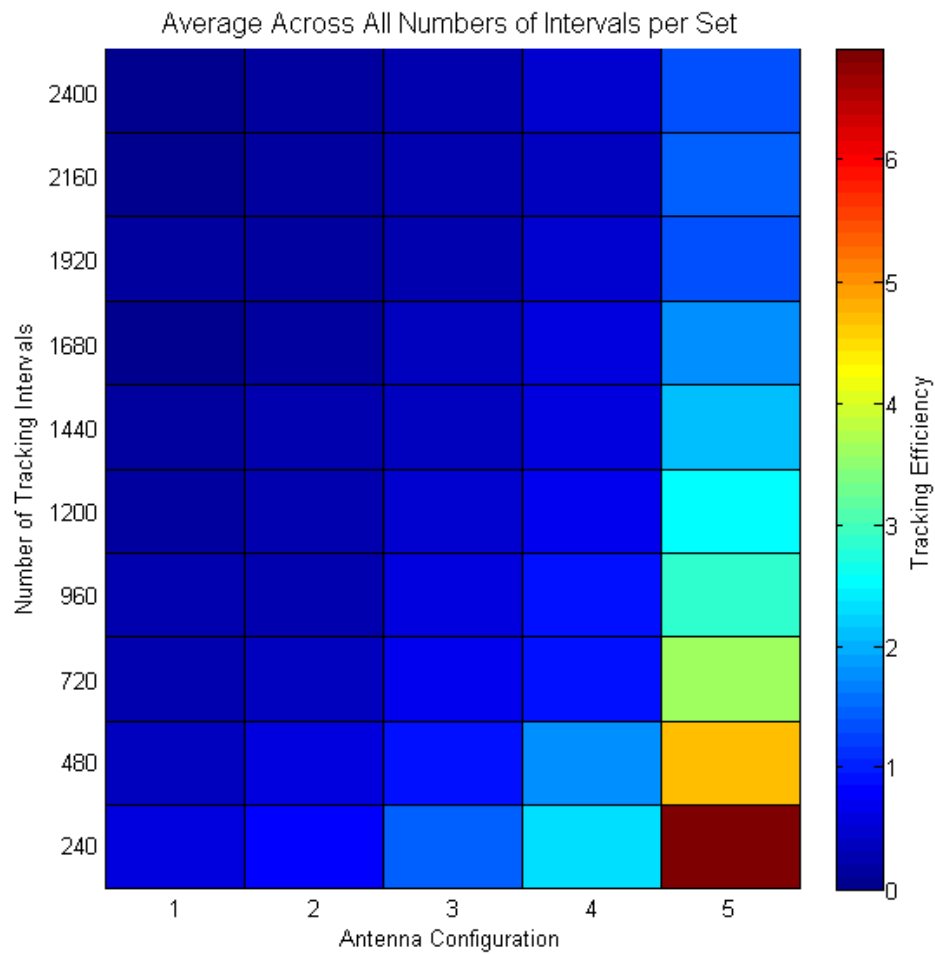


Figure 4.20: Tracking Efficiency vs. Antenna Configuration vs. Number of Tracking Intervals Averaged Across All Numbers of Intervals per Set

Differences Across Presented Examples

A mission designer may also be able to derive strategies for selecting cost-effective tracking solutions by comparing the most efficient strategy for each example. If the most cost-effective solution is the same for all the examples, it may imply that cost-efficiency is not a function of the fixed variable. However, if the most cost-effective solution differs across the example, it may imply that cost-efficiency is a function of the fixed variable. A summary of the results of all eight examples tested in this case is shown in Table 4.1.

Table 4.1: Most Cost-Efficient Solution for All Tested Intervals per Set

| Intervals per Set | Antenna Configuration | Total Number of Tracking Intervals |
|--------------------------|------------------------------|---|
| 1 | 5 | 240 |
| 2 | 5 | 240 |
| 3 | 5 | 240 |
| 4 | 5 | 240 |
| 6 | 5 | 240 |
| 8 | 5 | 240 |
| 12 | 5 | 240 |
| 24 | 5 | 480 |

The most significant thing to note is that the most cost efficient tracking solution for each defined number of intervals per set is the same for 7 of the 8 options. When holding the grouping of the intervals constant, the most cost-efficient option for total number of tracking intervals is generally the same. Since the solution does not change as a function of the number of intervals per set, it first appears that the grouping of the intervals does not significantly affect the performance of a given tracking solution.

However, the most cost-efficient solution for 24 intervals per set differs from the rest, with 480 tracking intervals being the best option as opposed to 240 tracking intervals. It is hypothesized that this is related to the length of the gaps in coverage in the schedules. For a

defined number of intervals per set, as the total number of intervals decreases, the number of sets decreases as well. Less tracking sets corresponds to longer gaps in coverage. With 240 tracking intervals grouped in sets of 24, there are 10 sets evenly spaced across the mission. With 480 tracking intervals grouped in sets of 24, there are 20 sets. Though it is not proven in this analysis, it is suspected that the decrease in uncertainty due to having 20 sets (versus 10 sets) evenly spaced across the mission is significant enough that it justifies the cost of the 240 extra tracking intervals required.

The existence of the discrepancy in the most cost-efficient total number of tracking periods for 24 intervals per sets indicates that the grouping of intervals may, in fact, affect the performance of a given tracking solution. This is further indicated by the fact that the solutions for the other seven values of intervals per set use the minimum tested value of total number of intervals. This may indicate that the tested values of total number of intervals did not go low enough to show the effect on cost-efficiency of changing the value of intervals per set. As the tested total numbers of tracking intervals were selected arbitrarily, so this is possible.

As the total number of intervals was not tested lower than 240, this hypothesis cannot be proven using the data collected. Additionally, there are not enough data points to definitively show a trend in the cost efficiencies. In order to test this hypothesis, the analysis in this thesis would have to be re-run for lower numbers of tracking intervals.

Additional Considerations for Mission Designer

This case highlights a weakness of the method outlined in this thesis. Some patterns and trends that are important to the mission designer may not be visible if too narrow of a range is used for the tested values of a variable. On the other hand, however, testing too wide of a range may take a very long time. As this method is meant to offer the mission designer a relatively

quick rough order of magnitude estimate of the schedule requirements, it may not be worth the time to run a long, intensive analysis. It is recommended that a mission designer arbitrarily select a moderately wide range of values to test, and if a potential pattern is seen towards one edge of the range, the range can be expanded and retested as necessary.

Summary

In summary, in the event that a number of intervals per set is defined, mission designers can select the most cost-efficient tracking solution by testing all the possible solutions with that number of intervals per set. For this example, there are five antenna configurations and ten values for total number of tracking intervals, so there are fifty solutions to test. However, a mission designer can select as many options for each of these variables as desired. The most cost-efficient tracking solution of the choices tested is the option with the highest tracking efficiency value.

In the event that the mission designer desires to use a different solution than the most cost-efficient option, they can look at the relative performance of all the solutions for a defined number of intervals per set to identify patterns that allow them to make cost-efficient decisions. For this example, it is seen from the results that the cost of using more than one 34-meter antenna per site is generally not worth the extra accuracy if selecting based on cost-efficiency. Additionally, for most of the values of intervals per set tested, it appears that there is a minimum number of total tracking solutions that offers the optimum trade-off between tracking cost and achieved accuracy. A mission designer generating a tracking schedule for this mission would be able to see that, if they decided to deviate from the most cost-efficient solution, they will achieve the best performance on average by using Antenna Configuration 5 and/or 240 total tracking intervals.

Case 3: Defined Antenna Configuration

This case discusses the process for a scenario where the antenna configuration is defined. Therefore, this case is an optimization problem across two variables (total number of tracking intervals and number of intervals per set) with one hard constraint (antenna configuration).

Process to Identify Most Cost-Efficient Solution

Five different examples of this case are presented, one for each of the possible antenna configurations considered in this thesis. The first example is using Antenna Configuration 1. Tracking simulations for every potential tracking solution using this antenna configuration are run. As there are ten different values for total number of tracking intervals and eight different values for intervals per set that can be used with the antenna configuration, eighty different solutions must be tested. Then, the tracking efficiency of all the potential solutions are compared. As in Case 1, the results must be shown three-dimensionally, and heat maps are again used. The results of this example are presented in Figure 4.21.

As in the charts in Case 1, each section of the heat map corresponds to a different potential solution. For this case, however, each row corresponds to a total number of tracking intervals, and each column corresponds to a number of intervals per set. The color of each section corresponds to the tracking efficiency achieved. The most cost-efficient tracking solution can be identified by selecting the section with the highest tracking efficiency. For this particular trajectory and antenna configuration, the solution using 240 intervals grouping the intervals in sets of 1 offers the highest tracking efficiency.

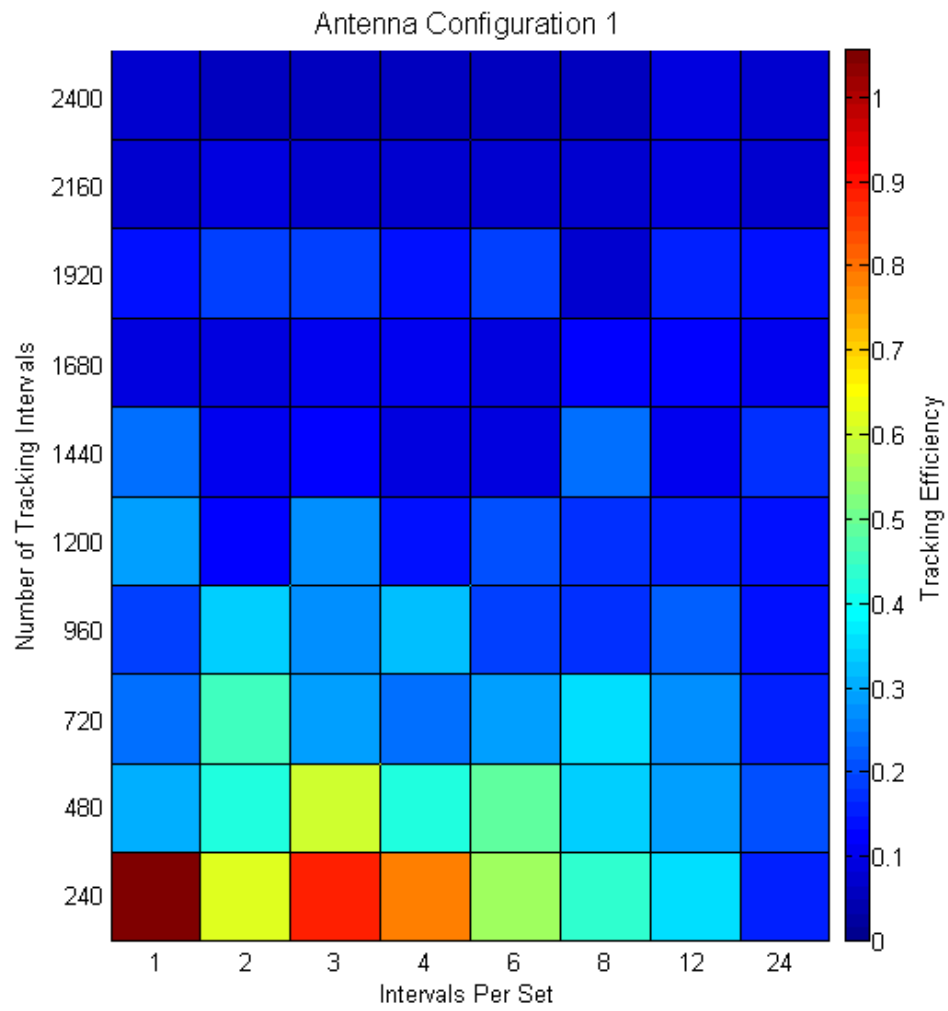


Figure 4.21: Tracking Efficiency vs. Number of Tracking Intervals vs. Number of Intervals per Set for Antenna Configuration 1

The second example is using Antenna Configuration 2. The process for this example is the same as the process for the first. There are again eighty possible solutions tested, their results plotted, and then the section with the highest tracking efficiency is selected. The results are presented in Figure 4.22. For this particular trajectory and antenna configuration, the solution using 240 total tracking periods grouped in sets of 1 offers the highest tracking efficiency.

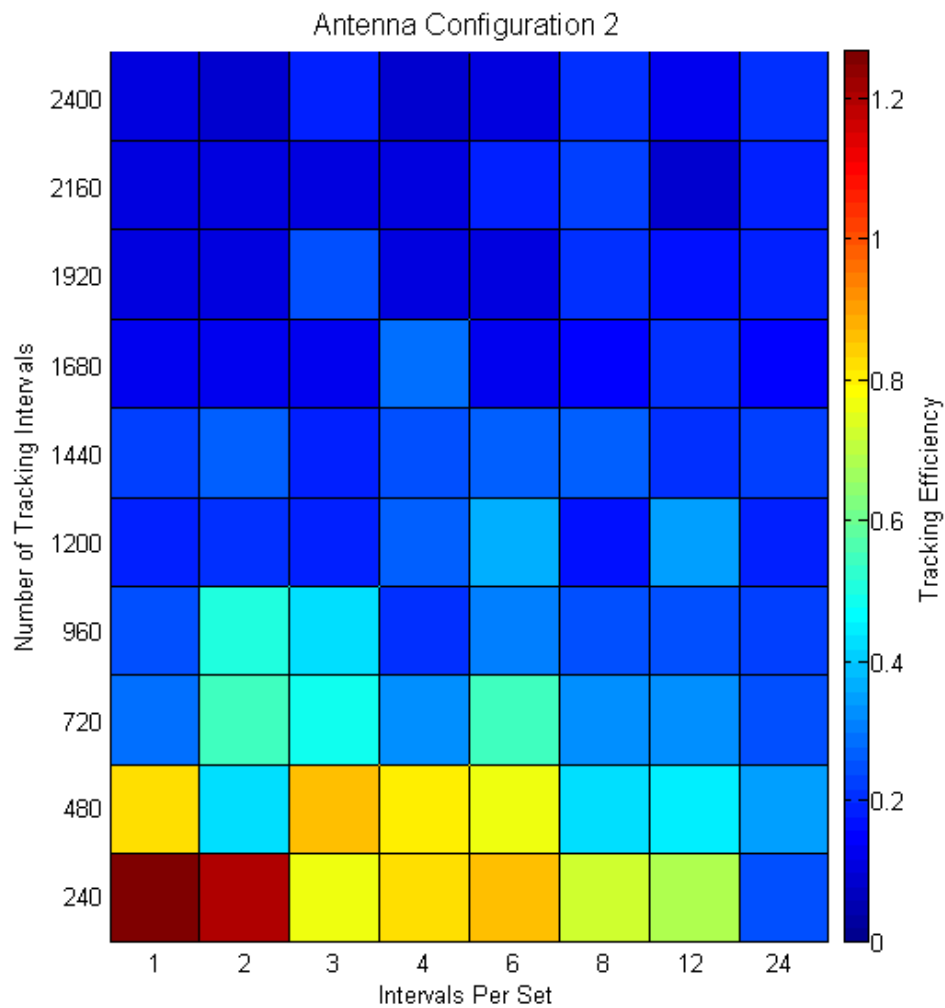


Figure 4.22: Tracking Efficiency vs. Number of Tracking Intervals vs. Number of Intervals per Set for Antenna Configuration 2

The third example is using Antenna Configuration 3. The results are presented in Figure 4.23. For this particular trajectory and antenna configuration, the solution using 240 total tracking periods grouped in sets of 1 offers the highest tracking efficiency.

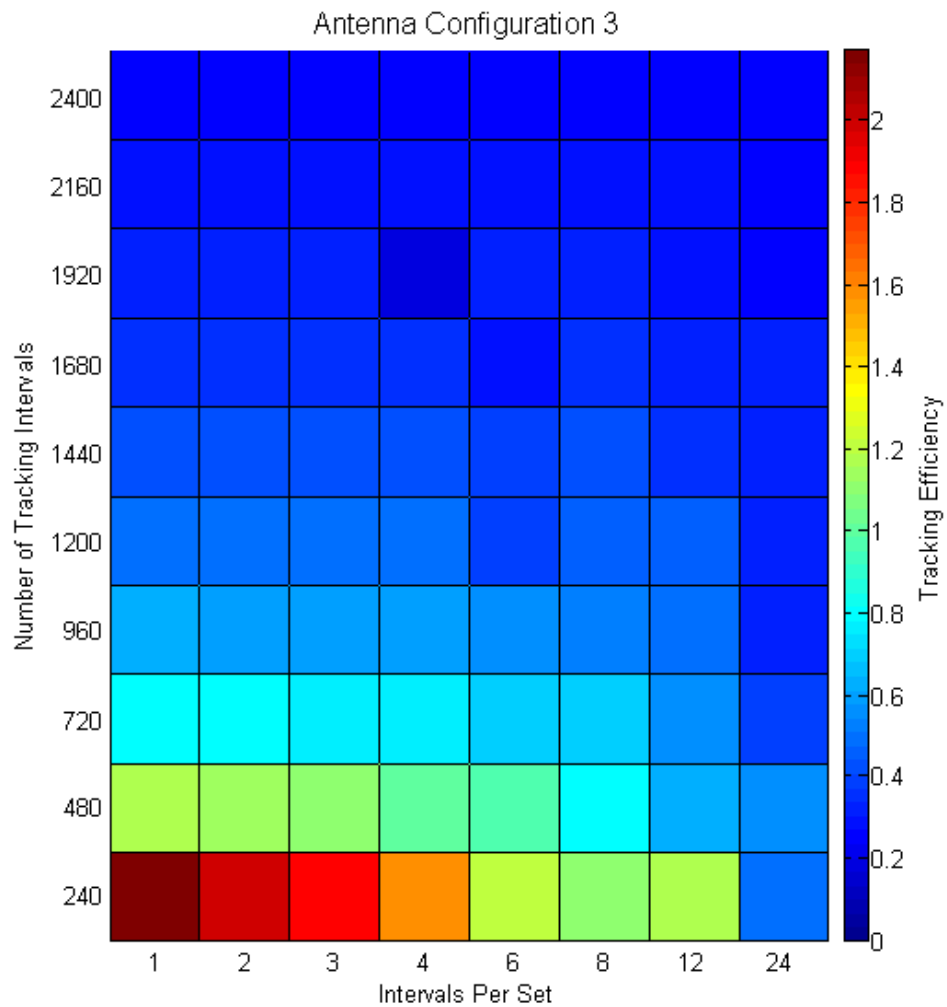


Figure 4.23: Tracking Efficiency vs. Number of Tracking Intervals vs. Number of Intervals per Set for Antenna Configuration 3

The fourth example is using Antenna Configuration 4. The results of this example are presented in Figure 4.24. For this example, the solution using 240 total tracking periods grouped in sets of 2 offers the highest tracking efficiency.

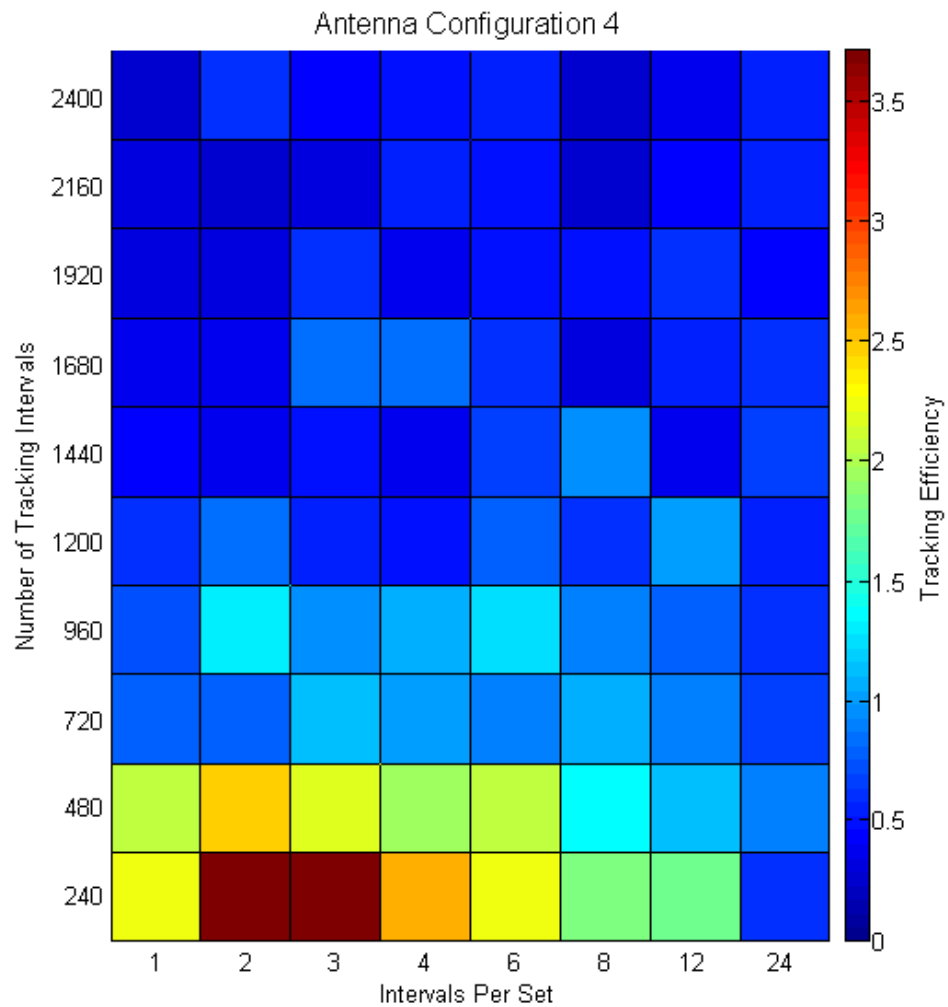


Figure 4.24: Tracking Efficiency vs. Number of Tracking Intervals vs. Number of Intervals per Set for Antenna Configuration 4

The fifth example is using Antenna Configuration 5. The results are presented in Figure 4.25. For this particular trajectory and antenna configuration, the solution using 240 total tracking periods grouped in sets of 1 offers the highest tracking efficiency.

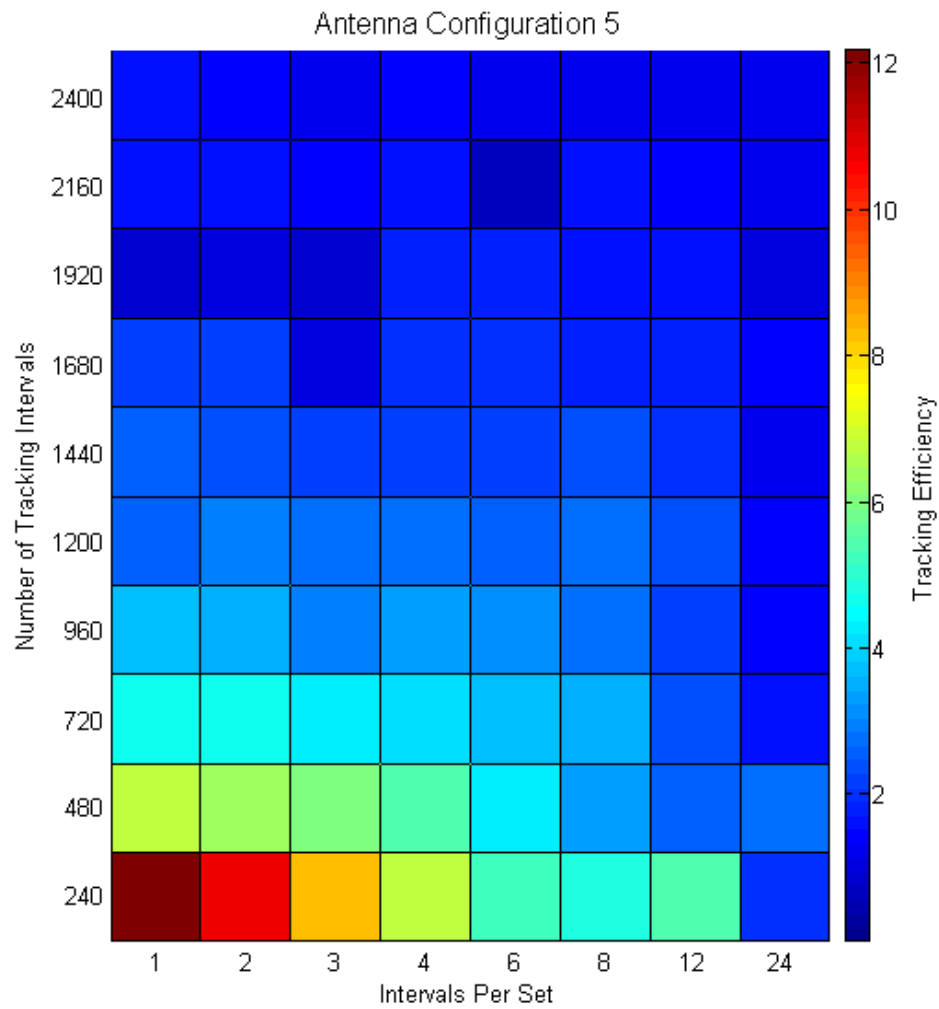


Figure 4.25: Tracking Efficiency vs. Number of Tracking Intervals vs. Number of Intervals per Set for Antenna Configuration 5

Sensitivity of Solution to Variables

For this sample trajectory, and Antenna Configuration 3, the most cost-efficient solution uses 240 total tracking intervals grouped in sets of 1. To determine the sensitivity of the cost-efficiency relative to each of the independent variables, the average performance of each column can be compared to the average performance of each row. The dimension with the higher average performance represents the variable to which the cost-efficiency is most sensitive. As can be seen in Figure 4.23, the average tracking efficiency across the solutions in the “240 Total Tracking Intervals” row is higher than the average across the solutions in the “1 Interval per Set” column. If a mission designer is forced to decide between two solutions: one using 240 total tracking intervals and not 1 interval per set, or one using 1 interval per set and not 240 total tracking intervals, they can look at the results and determine that, for a constant antenna configuration, the tracking efficiency is most strongly related to the number of tracking intervals. The designer should select the option using 240 tracking intervals.

Robustness of Solution

Another analysis can be done using the results of the examples above to estimate the robustness of the selected solution. All of the figures above display the performance of all the possible solutions for each fixed antenna configuration. However, there may be cases where the antenna configuration is still defined, but it is subject to change. If a mission designer wants to select a tracking solution that is robust to changes in the antenna configuration, they can average the results of each of the individual antenna configuration cases and identify the option with the best average performance.

For this sample trajectory, the results of this calculation are presented in Figure 4.26. The solution that is most robust to changes in the antenna configuration is found by selecting the section with the highest average tracking efficiency across all the cases. In this case, the most robust solution uses 240 tracking intervals grouped in sets of 1.

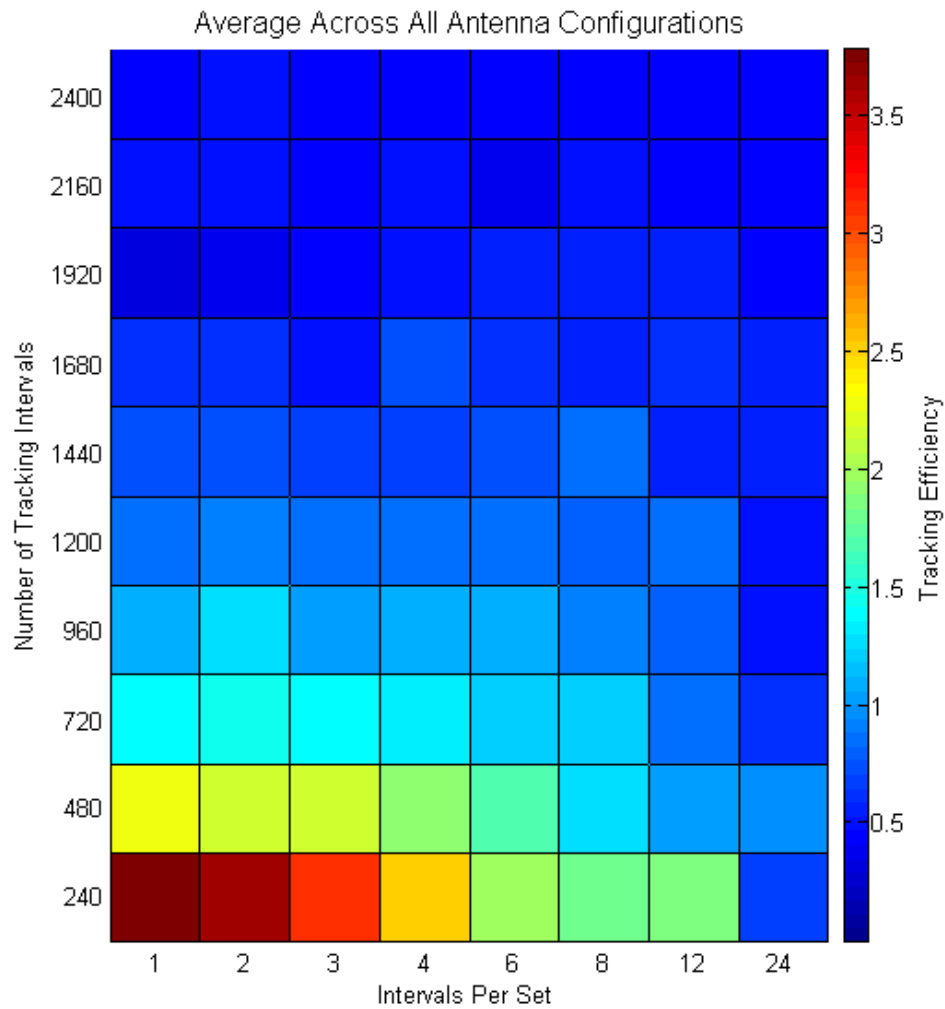


Figure 4.26: Tracking Efficiency vs. Number of Tracking Intervals vs. Number of Intervals per Set Averaged Across All Antenna Configurations

Summary

In summary, in the event that the antenna configuration is defined, mission designers can select the most cost-efficient tracking solution by testing all the possible solutions with that configuration. For this example, there are eight values for intervals per set and ten values for total number of tracking intervals, so there are eighty solutions to test. However, a mission designer can select as many options for each of these variables as desired. The most cost-efficient tracking solution of the choices tested is the option with the highest tracking efficiency value.

In the event that the mission designer desires to use a different solution than the most cost-efficient option, they can look at the relative performance of all the solutions for a defined number of intervals per set to identify patterns that allow them to make cost-efficient decisions. For this example, it is seen from the results that the cost of using more than one 34-meter antenna per site is generally not worth the extra accuracy if selecting based on cost-efficiency. Additionally, for most of the antenna configurations tested, it appears that there using a low number of tracking intervals evenly spaced is a strategy to achieving cost-efficiency. However, it is hypothesized that too few tracking intervals will offer less performance. A mission designer generating a tracking schedule for this mission would be able to see that, if they decided to deviate from the most cost-efficient solution, they will achieve the best performance on average by using 240 tracking intervals and/or 1 interval per set.

Additionally, if the mission designer desires to identify a solution that is most robust to changes in the antenna configuration (for example, in case some of the antennas are diverted to other spacecraft), they can select it using this method. By determining the average performance across all antenna configurations, the solution most robust to these changes can be found by selecting the option with the highest efficiency.

Case 4: No Constraints

This case discusses the process for a scenario where there are no restrictions and nothing predefined. Therefore, this case is an optimization problem across three different variables (total number of tracking intervals, number of intervals per set, and antenna configuration) with no constraints.

Process to Identify Most Cost-Efficient Solution

Tracking simulations for every potential tracking solution are run. As there are ten different values for total number of tracking intervals, eight different values for intervals per set, and five different antenna configurations, 400 different solutions must be tested. Then, the tracking efficiency of all the potential solutions are compared. The results of this process are presented in Figure 4.27.

As opposed to previous cases, the results of Case 4 must be shown four-dimensionally. To display this, a plot of multiple heat maps are used. Each heat map corresponds to one antenna configuration, and is constructed the same way as the heat maps in Case 2. A common color scale to display tracking efficiency is used across all the heat maps. This allows for comparison of tracking efficiency across all 400 potential solutions. The most cost-efficient tracking solution across all three variables is identified by selecting the section of all five plots with the highest tracking efficiency. For this example, a solution using 240 total tracking intervals, 1 interval per set, and Antenna Configuration 5 offers the highest tracking efficiency.

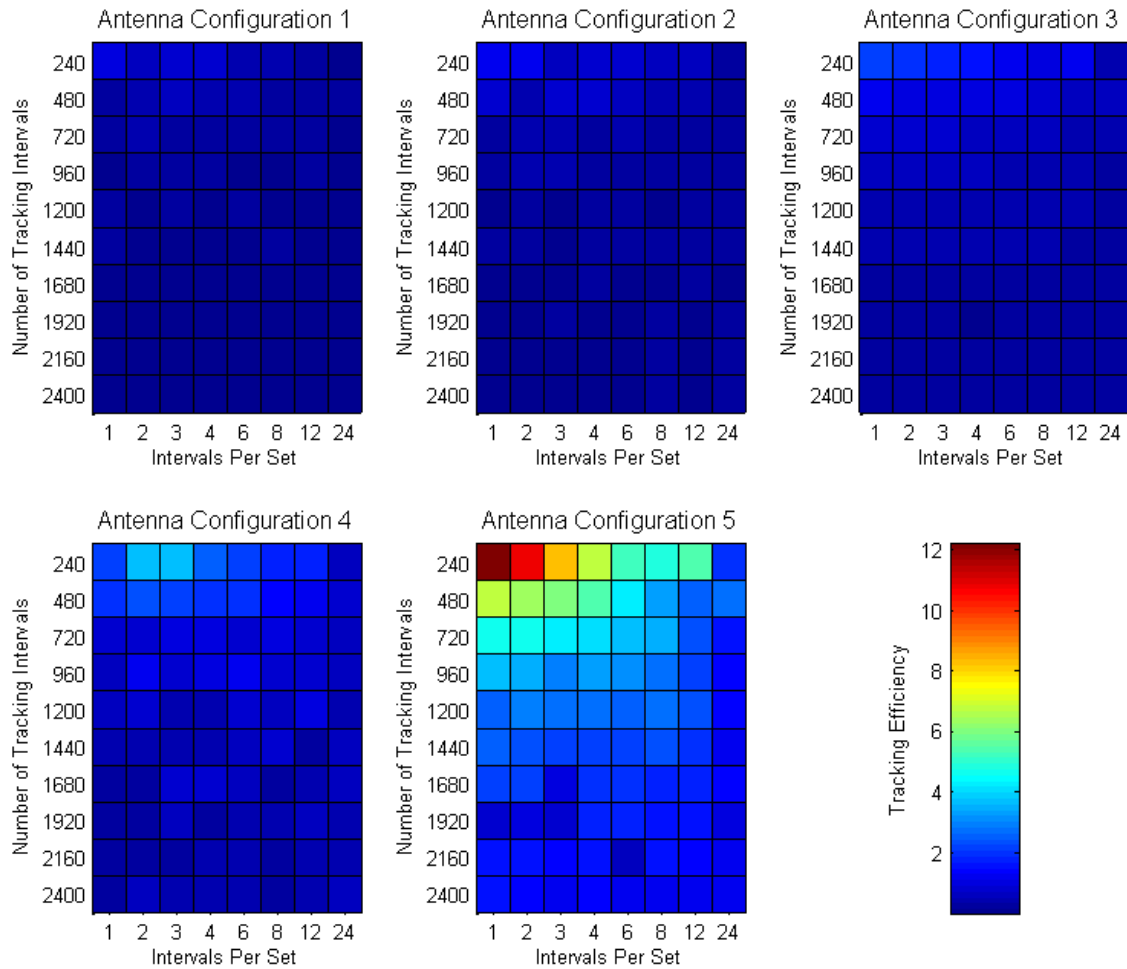


Figure 4.27: Tracking Efficiency by Antenna Configuration Across Several Different Tracking Schedules (No Constraints)

Sensitivity of Solution to Variables

In this case, several of the patterns seen in earlier cases are also seen here. Generally, it is seen that the average cost-efficiency of solutions using Antenna Configuration 5 is higher than the average for any of the other configurations. Additionally, most of the best solutions are in the low tracking intervals/low intervals per set section of the plots. For most of the values of intervals per set tested, it appears that there is a minimum number of total tracking solutions that offers the optimum trade-off between tracking cost and achieved accuracy. A mission designer generating a

tracking schedule for this mission would be able to see that, if they decided to deviate from the most cost-efficient solution, they will achieve the best performance on average by using Antenna Configuration 5, 240 total tracking intervals, and/or 1 interval per set.

Summary

In summary, in the event that there are no constraints, mission designers can select the most cost-efficient tracking solution by testing all the possible solutions. For this example, there are five antenna configurations, eight values for number of intervals per set, and ten values for total number of tracking intervals, so there are four hundred solutions to test. However, a mission designer can select as many options for each of these variables as desired. The most cost-efficient tracking solution of the choices tested is the option with the highest tracking efficiency value.

In the event that the mission designer desires to use a different solution than the most cost-efficient option, they can look at the relative performance of all the solutions for a defined number of intervals per set to identify patterns that allow them to make cost-efficient decisions. For this example, it is seen from the results that Antenna Configuration 5 offers the largest selection of cost-efficient options, indicating that the cost of using more than one 34-meter antenna per site is generally not worth the extra accuracy if selecting based on cost-efficiency. This indicates that cost-efficiency is most sensitive to the antenna selection. By looking at the results further, it is seen that cost-efficiency is second most sensitive to the number of tracking intervals and least sensitive to the number of intervals per set. A mission designer generating a tracking schedule for this mission would be able to see that, if they decided to deviate from the most cost-efficient solution, they will achieve the best performance on average by using Antenna Configuration 5, 240 total tracking intervals, and/or 1 interval per set.

Case 5: Defined Maximum Covariance Requirement

This case discusses the process for a scenario where there are no restrictions on tracking schedule or antenna configuration, but there is a maximum covariance requirement. Therefore, this case is an optimization problem across three different variables (total number of tracking intervals, number of intervals per set, and antenna configuration) with one soft constraint (maximum covariance).

Process to Identify Most Cost-Efficient Solution

This case is different from the other cases shown so far because, as opposed the previous cases where the constraint is on the independent variables, the constraint in this case is on the dependent variable. The process for this case begins by running tracking simulations for every possible tracking solution, as in Case 3. Once the results are found, all of the solutions that do not meet the maximum covariance requirement are removed. Then, the tracking efficiency of all the remaining solutions are compared.

For the example presented here, a maximum average covariance of 1500 is arbitrarily defined as the requirement. The first step, the removal of any test cases with an average covariance higher than the defined requirement, is shown in Figure 4.28. On the plot, every point represents one tracking solution, and they are shown versus their average covariance values. A red 'X' on the plot indicates a test case that was rejected and removed from the data set because it did not meet the covariance requirement.

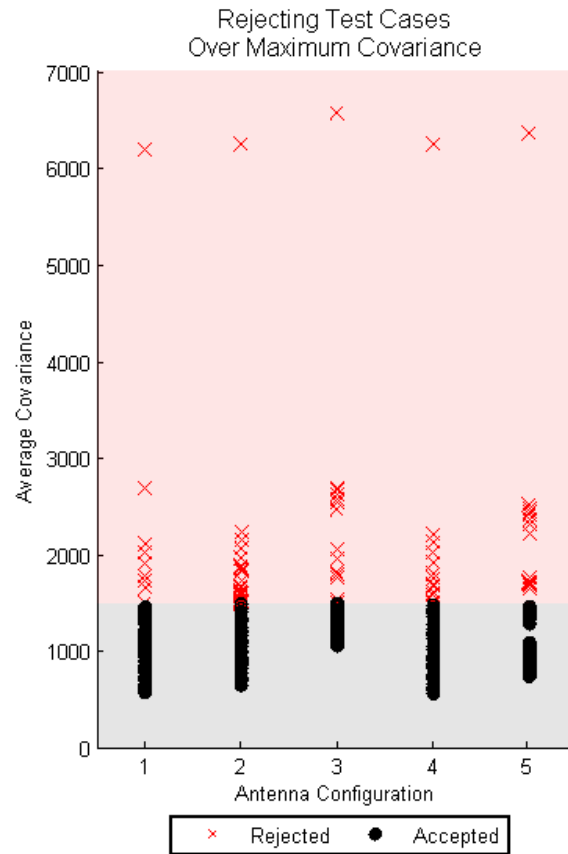


Figure 4.28: Average Covariance vs. Antenna Configuration (Covariance Limited)

The second step is to compare the tracking efficiencies of the remaining solutions. All of the cases are shown on heat maps using the same tracking efficiency color scale. These results are shown in Figure 4.29. The crossed-out sections are the solutions removed in the first step of this process, and the color of those sections is set to black for extra clarity when looking at the plots.

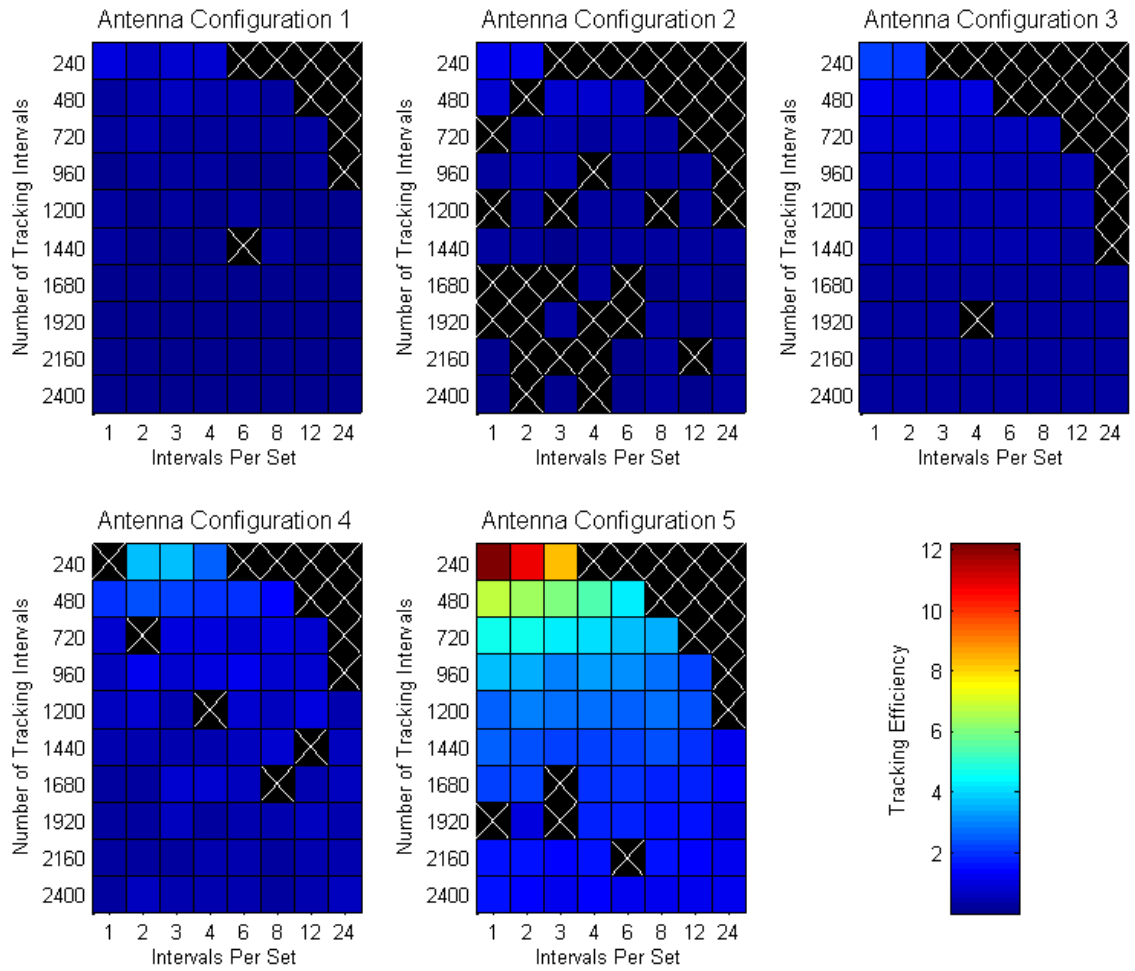


Figure 4.29: Tracking Efficiency by Antenna Configuration Across Several Different Tracking Schedules (Covariance-Limited), where a crossed out section indicates a rejected solution option

The most cost-efficient option of the remaining tracking solutions is identified by selecting the section from all five plots with the highest tracking efficiency. For this example, the solution using 240 total tracking intervals, 1 interval per set, and Antenna Configuration 5 offers the highest tracking efficiency.

Sensitivity of Solution to Variables

For this sample trajectory, and a maximum covariance, the most cost-efficient solution uses Antenna Configuration 5, 240 total tracking intervals, and intervals grouped in sets of 1. To determine the sensitivity of the cost-efficiency relative to each of these independent variables, the average performance relative to each variable can be compared. The variable with the higher average performance represents the variable to which the cost-efficiency is most sensitive. As can be seen in Figure 4.29, the average tracking efficiency across the remaining solutions in the “Antenna Configuration 5” section is higher than the average across the solutions in any of the other antenna configurations. The sensitivity of cost-efficiency is most strongly related to the antenna configuration selected. For this particular case, the cost-efficiency of the remaining solutions is most strongly related to the total number of tracking intervals, as evidenced by the fact that the average performance by row is higher than the average performance by column. When selecting a tracking option for this case, the mission designer should prioritize the use of Antenna Configuration 5, as that is the variable most strongly affecting cost-efficiency.

Case 6: Defined Maximum Cost

This case discusses the process for a scenario where there are no restrictions on tracking schedule or antenna configuration but there is a maximum cost requirement. Therefore, this case is an optimization problem across three different variables (total number of tracking intervals, number of intervals per set, and antenna configuration) with one soft constraint (maximum cost). Cost is assumed to be directly proportional to the weighted number of observations, so the constraint in this problem is applied based on the weighted measurement count.

Process to Identify Most Cost-Efficient Solution

As in Case 4, the constraint is on a dependent variable. This process consists of first testing all combinations of variables, as in Case 3. Once the results are found, all of the solutions that do not meet the maximum cost requirement are removed. The remaining solutions are then compared, and the option with the highest tracking efficiency is identified.

For the example presented here, a maximum weighted observation count of 20,000 is arbitrarily defined as the requirement. The first step, the removal of any test cases with a weighted observation count higher than the defined requirement, is shown in Figure 4.30. On the plot, every point represents one tracking solution, and they are shown versus their weighted observation counts. A red 'X' on the plot indicates a test case that was rejected and removed from the data set because it did not meet the cost requirement.

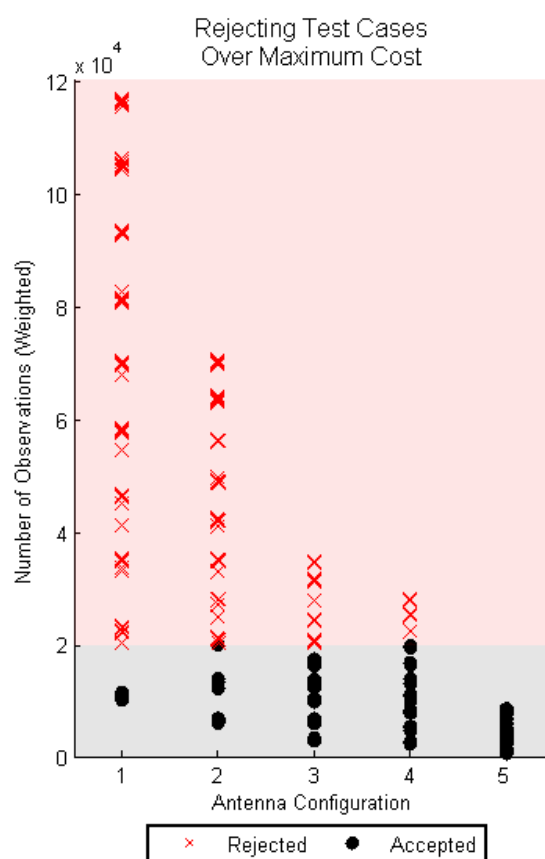


Figure 4.30: Number of Observations vs. Antenna Configuration (Cost-Limited)

The second step is to compare the tracking efficiencies of the remaining solutions. All of the cases are shown on heat maps using the same tracking efficiency color scale. These results are shown in Figure 4.31. The crossed-out sections are the cases removed in the first step of this process, and the color of those sections is set to black for extra clarity when looking at the plots.

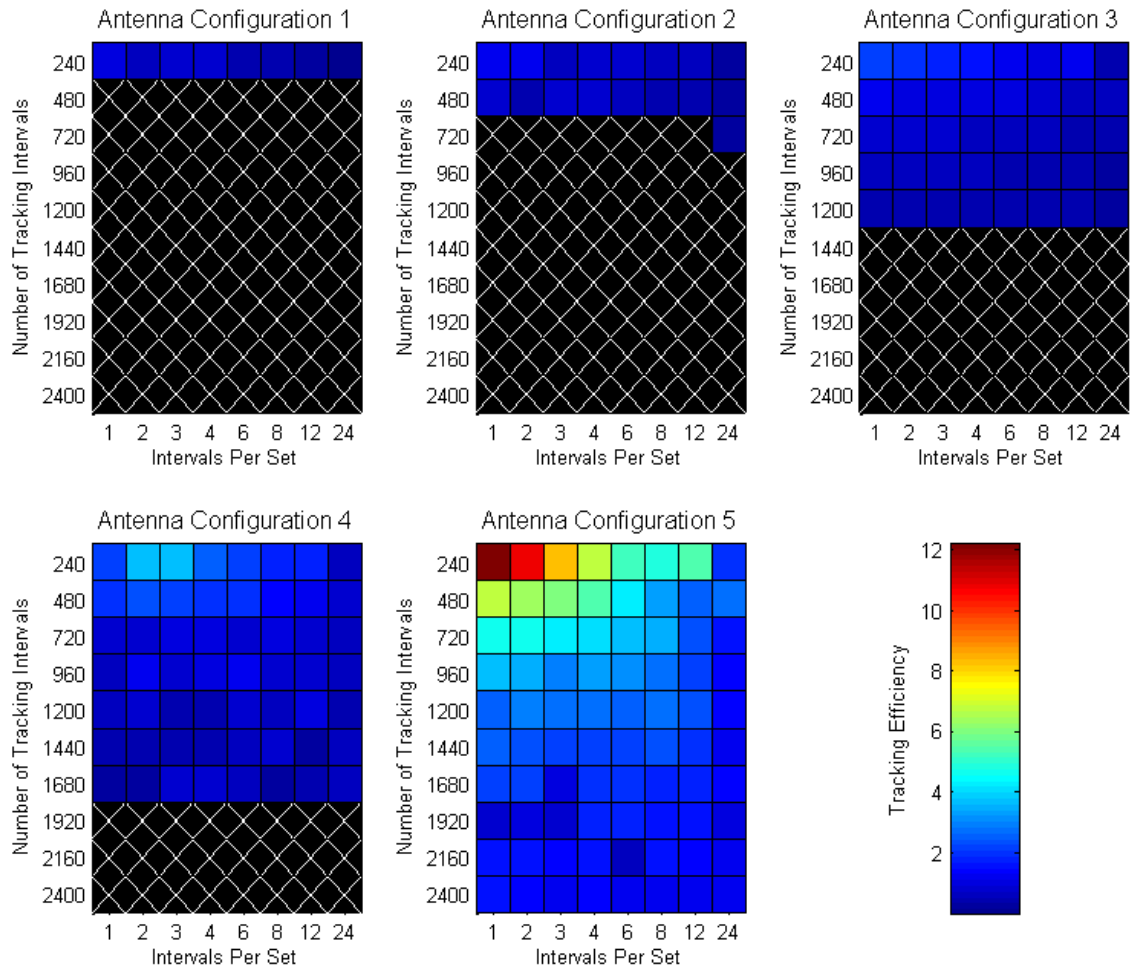


Figure 4.31: Tracking Efficiency by Antenna Configuration Across Several Different Tracking Schedules (Cost-Limited), where a crossed out section indicates a rejected solution option

The most cost-efficient option of the remaining tracking solutions can be identified by selecting the section from the five plots with the highest tracking efficiency. For this example, the solution using 240 total tracking intervals, 1 interval per set, and Antenna Configuration 5 offers the highest tracking efficiency.

Sensitivity of Solution to Variables

For this sample trajectory, and a maximum cost, the most cost-efficient solution uses Antenna Configuration 5, 240 total tracking intervals, and intervals grouped in sets of 1. To determine the sensitivity of the cost-efficiency relative to each of these independent variables, the average performance relative to each variable can be compared. The variable with the higher average performance represents the variable to which the cost-efficiency is most sensitive. As can be seen in Figure 4.31, the average tracking efficiency across the remaining solutions in the “Antenna Configuration 5” section is higher than the average across the solutions in any of the other antenna configurations. The sensitivity of cost-efficiency is most strongly related to the antenna configuration selected. For this particular case, the cost-efficiency of the remaining solutions is most strongly related to the total number of tracking intervals, as evidenced by the fact that the average performance by row is higher than the average performance by column. When selecting a tracking option for this case, the mission designer should prioritize the use of Antenna Configuration 5, as that is the variable most strongly affecting cost-efficiency.

Case 7: Defined Maximum Cost and Maximum Gap in Coverage Duration

This case discusses the process for a scenario where there are no restrictions on tracking schedule or antenna configuration, but there is both a maximum cost and a maximum gap in coverage duration requirement. Therefore, this case is an optimization problem across three different variables (total number of tracking intervals, number of intervals per set, and antenna configuration) with two soft constraints (maximum cost and maximum gap in coverage duration).

As in Cases 5 and 6, the constraints are on a dependent variable. Therefore, the process for this case is mostly the same as the process in Case 5 and Case 6. However, because there are two constraints on dependent variables instead of just one, the process is slightly different.

For the example presented here, a maximum gap in coverage length of 500 seconds is arbitrarily defined as the gap length requirement and a maximum cost of 20,000 is arbitrarily defined as the cost requirement. The first step, the removal of any test cases that don't meet the maximum cost requirement or maximum gap length requirement, is shown in Figure 4.32, where the plot on the right shows a zoomed-in view of the accepted region of the plot on the left. A red 'X' on the plot indicates a test case that was rejected and removed from the data set because it did not meet the requirement. There are two overlapping rejection regions seen on the plot that correspond to each of the two requirements.

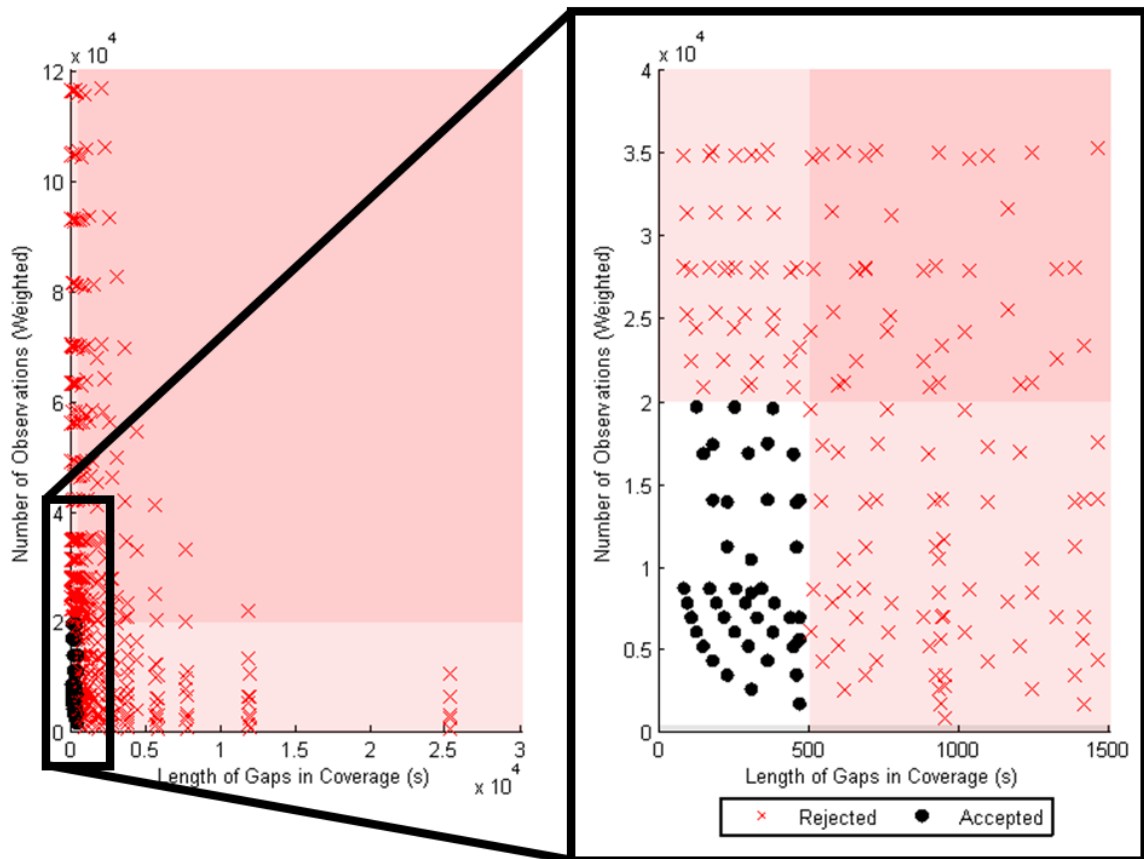


Figure 4.32: Number of Observations (Weighted) vs. Length of Gaps in Coverage (Cost and Gap Duration Limited)

The second step is to compare the tracking efficiencies of the remaining solutions. All of the cases are shown on heat maps using the same tracking efficiency color scale. These results are shown in Figure 4.33. The crossed-out sections are the solutions removed in the first step of this process, and the color of those sections is set to black for extra clarity when looking at the plots.

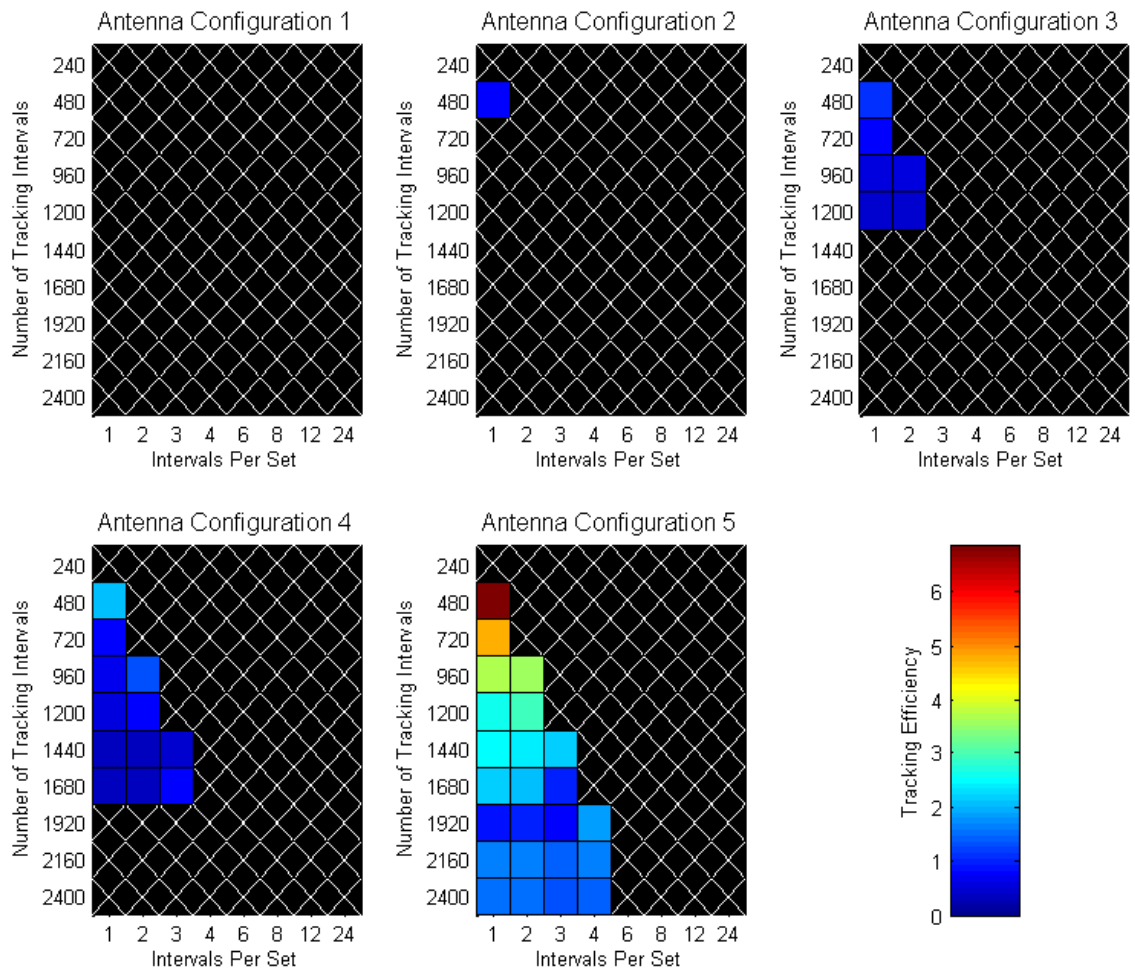


Figure 4.33: Tracking Efficiency by Antenna Configuration Across Several Different Tracking Schedules (Cost and Gap Duration Limited), where a crossed out section indicates a rejected solution option

The most cost-efficient option of the remaining tracking solutions can be identified by selecting the section from the five plots with the highest tracking efficiency. For this example, the solution using 480 total tracking intervals, 1 interval per set, and Antenna Configuration 5 offers the highest tracking efficiency.

Sensitivity of Solution to Variables

For this sample trajectory, and a maximum cost and maximum gap in coverage duration length, the most cost-efficient solution uses Antenna Configuration 5, 480 total tracking intervals, and intervals grouped in sets of 1. To determine the sensitivity of the cost-efficiency relative to each of these independent variables, the average performance relative to each variable can be compared. The variable with the higher average performance represents the variable to which the cost-efficiency is most sensitive. As can be seen in Figure 4.33, the average tracking efficiency across the remaining solutions in the “Antenna Configuration 5” section is higher than the average across the solutions in any of the other antenna configurations. The sensitivity of cost-efficiency is most strongly related to the antenna configuration selected. Also, for this particular case, the cost-efficiency of the remaining solutions is most strongly related to the number of intervals per set, as evidenced by the fact that the average performance by column is higher than the average performance by row. When selecting a tracking option for this case, the mission designer should prioritize the use of Antenna Configuration 5, as that is the variable most strongly affecting cost-efficiency.

Additional Considerations for Mission Designer

The way each tracking schedule is created in the analysis, every gap in coverage is the same length. However, if schedules are generated in a different way, where gaps in coverage are of variable length, an additional step is required to search through the generated tracking schedule and identify the length of the longest gaps in coverage.

Chapter 5

Conclusions and Future Work

In this thesis, methods for identifying cost-efficient tracking solutions were demonstrated for an Earth-Moon trajectory under a variety of different constraints. Though the specific results presented in this thesis are not applicable to any mission but the one used here, the process can be applied to a wide range of interplanetary missions. The results of the process meet the research goals. They are able to provide the mission designer with information about the sensitivity of cost-efficiency to the independent variables. They can be used to determine the most robust solution to unexpected changes in the constrained variables. They can show patterns in the data that inform the mission designer about trade-offs in selection of the independent variables and will assist in defining a starting point and a search direction for more detailed and configurable searches. Overall, this process provides a valuable tool for any mission designers seeking to generate a cost-efficient tracking schedule.

Several options exist for further work. First, the fidelity of the cost model can be increased. An expanded cost calculation for use of the DSN exists that provides an even more accurate estimate of the cost, though it includes other variables that were not covered in this thesis, including costs of calibrating the DSN before every observation. In order to generate a more accurate estimate of the cost, these additional variables should be incorporated into the calculation of the tracking efficiency. This expanded cost calculation can be found in Reference 6.

Furthermore, the tracking schedule representation used in this thesis offers a wide variety of options for generated schedules, though there are other ways of defining the schedule that would offer even more schedule options. For example, a binary string where every element indicates whether tracking is on or off for a period of time would be an extremely configurable

way to represent a tracking schedule. This representation would be also be well suited for a method of identifying cost-effective solutions using evolutionary strategies.

Additionally, this thesis does not include methods of cost saving through the use of antenna arraying. The DSN includes the capability to array multiple smaller dishes together to achieve similar, or better, performance than using a single larger antenna. This analysis did not consider these cases, even though they may offer cost savings, especially with missions that require the use of the bigger, 70-meter dishes.

Finally, the methods shown in this thesis are only for single spacecraft tracking. They do not provide an overall cost-efficient tracking solution for tracking multiple spacecraft at once. For future analysis, it would be useful to consider the requirements of multiple spacecraft together to determine the most cost-efficient solution for the entire set of missions.

All of these opportunities for future work will allow the results to be generalizable to a wider variety of situations. However, the methods shown in this thesis are sufficient to provide rough order of estimate values for the performance of different tracking solutions across a considerable range. The methods shown in this thesis can assist mission designers in identifying cost-effective tracking solutions for a large variety of spacecraft under several different constraints.

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